Money Demand, a Microeconometric---Seminonparametric Approach: The Asymptotoically Ideal Model (AIM)

Darron Thomas School of Business Administration, University of Technology Jamaica 237 Old Hope Road, Papine, Kingston 6 Jamaica W. I. E-mail: darron.thomas@gmail.com

Abstract

The Asymptotically Ideal Model (AIM), first estimated by Barnett and Yue (1998), based on the Muntz-Szatz series expansion as described by Barnett and Jonas (1983), is used to estimate money demand using quarterly US data from 1960 to the first quarter of 2004. We find that monetary assets are generally substitutes and that the unitary income elasticity postulate is satisfied. Unfortunately though, we are unable to resolve the debate as to whether or not money demand is a stable process. We are also not able to test the money neutrality hypothesis in our model, but a clear way on how to do this is suggested for the purpose of future research.

Keywords: Money Demand, Monetary Assets, Asymptotically Ideal Model, Seemingly Unrelated Regression, Semi-nonparametric.

JEL Classification: C13, C14, C40, E41.

Introduction

This paper estimates a money demand function based on the microeconomic foundations of the money demand process using quarterly US data from 1960 to 2004. Such a contribution is necessary as the standard approaches have fallen short regarding the explanation of major phenomena and experience. Some of these short comings are evidenced by the general inability to explain the behavior of the money demand process in anomalous periods such as "the case of the missing money", "the great velocity decline", "the M1 explosion" and, of course, the German hyperinflations of the 1920's. A notable exception to these generally failed models is the work of Baba, Hendry and Starr (BHS) (1992), who have been able to explain the first three above mentioned anomalies. Another exception to the rule is Michael, Nobay and Peel (1994) which provides an explanation of the behavior of the money demand process during the German hyperinflation. For expositions employing the standard approach to estimating money demand, which is usually in the cointegration tradition, see Samreth (2008) as well as Bashier and Dahlan (2011). In more recent times, there have been panel approaches to estimating money demand such as Narayan (2009).

Issues regarding money demand have long been at the forefront of economic thinking (see for example Friedman (1956)). However, even the most well educated debates and empirical investigations continue to be eluded by this very important aspect of the functioning of the economy. Debates of money demand often focus on a structural aggregate representation of the process, and usually posit that it is a function of income, interest rates and prices, among other determinants. This has led to crude aggregated formulations of structural equations in most of the early attempts to capture the data generating mechanism underlying the money demand process. Unfortunately though, these models, with a few exceptions, as noted above have been unable to capture the patterns underlying the money demand process in what are generally considered anomalous periods.

In light of these failures along with the Lucas critique and a now generally accepted view that the microeconomic foundations should play a greater role in all economic discourse, there have been a number of attempts to estimate money demand functions based on the micro foundations underlying the process. Before moving on to the actual model that we estimate let us give a brief overview of the literature underlying the estimation of demand systems premised on microeconomic foundations. Of course, the main specifications of microeconomic expenditure/demand systems are the: (i) ALIDS model of Deaton and Muellbauer (1980), (ii) The Translog model of Christensen, Jorgensen and Lau (1975) and (iii) The Rotterdam model of Barnett (1979). In the case of (i) and (ii), which both have at the heart of their foundation a Taylor series expansion, it is widely known that these functional forms may not maintain their flexibility properties globally, or necessarily meet the regularity and/or other restrictions of utility maximizing behavior see Barnett (1983), Gallant (1981), Wales (1977) and White (1980).

In general, in these models there is a trade off between global regularity and global flexibility and in fact both may never be obtained simultaneously. The Rotterdam model, on the other hand, is not considered reasonable because it is highly restrictive in that it implies that the underlying utility function is either Cobb-Douglas or CES, and, thus, that the elasticities of substitution are constant.

As a consequence, the attempts to use microeconomic specifications have tried to make use of functional forms which are both globally flexible, separable and satisfy the conditions of utility maximizing behavior. A well known property of Fourier series expansions is that they converge to some continuous function and therefore can represent a nice utility function. This allowed, Fisher (1989) as well as Fisher and Fleissig (1994) to satisfy these conditions by estimating models based on Fourier series expansions, using US data. However, Barnett and Yue (1988) suggest that the trigonometric (sine and cosine profiles) cycles that characterize Fourier series expansions are not necessarily appropriate for economic data. Against this background, they propose using a Muntz-Szatz expansion, which in fact is globally flexible provided that all the models coefficients are positive, and calls it an Asymptotically Ideal Model (AIM). Yue (1991) uses the data from Fisher's work, which satisfies Varian's (1982) GARP and Separability tests, to estimate an AIM.

Yue (1991) estimates this model and creates series of the Allen partial elasticities by estimating these for each year in an attempt to capture the dynamics of the money demand process. Regretfully though, the model, while providing many new insights, still finds the case of 'the missing money' as well as other anomalous periods in money demand to be illusive. Though, a brilliant attempt, a number of short comings remain with the approach of Yue. The main shortcomings of Yue are addressed by Drake Fleissig and Swofford (DFS) (2003). These shortcomings are; firstly, it is a well established fact that money demand is a dynamic process and no attempt was made to capture this trait explicitly. Also, in the light of Blackorby and Russell (1981 and 1989), the Morishima elasticity of substitution is the more appropriate elasticity measure. However, Yue estimates the Allen Partial elasticities. Additionally, since we have a system of equations in addition to inequality restrictions on the parameters hypotheses test were not carried out.

Not withstanding their refinements, and especially so being the first attempt, DFS specified the dynamics inappropriately, thus concluding that the estimates were not invariant to the dropped share equation. This rendered their use of a model which may be particularly troublesome, in terms of biased estimates (Hendry and Mizon (1978)), because of the implied common factor restrictions imposed by the use of an AR(1) specification of the errors to capture the dynamics. It is these shortcomings that we wish to address in our discourse where an AIM will be used to estimate the expenditure system governing money Demand in the US. We intend to capture the dynamics of the money demand process by explicitly including the lags of the share equations as independent variables, Hendry and Mizon (1978), when we estimate the share equations. We will discuss these issues further as we proceed. The remainder of the article is organized as follows. Section 2 presents the model in its pure theoretical form. Section 3 discusses the data. Section 4 reports and discusses the estimates, while section 5 gives the conclusions.

Section 2: The Model

The Asymptotically Ideal Model (AIM) as proposed by Barnett and Jonas (BJ) (1983) is a Seminonparametric method as defined by Gallant (1981), and can be applied to the indirect utility function of a utility maximizing consumer facing a linear budget constraint. As is usually the case, due to homogeneity, in such a system we can normalize prices by expenditure in specifying the AIM. Commensurate with the fact that the Muntz - Szatz expansion can capture both global regularity and Global flexibility we can obtain arbitrarily accurate elasticity estimates over the entire space, rather than locally as obtains in the other specifications. In the work of BJ the Kth order Muntz – Szatz expansion is given as:

(1)
$$f(v) = a_0 + \sum_{k \in n} \sum_{i=1}^n a_k v_i^{\lambda(k)} + \sum_{k \in n} \sum_{m \in n} \left[\sum_{i=1}^n \sum_{j=1}^n a_{ijkm} v_i^{\lambda(k)} v_j^{\lambda(m)} \right] + \dots,$$

Where n is the number of goods and a_0 , a_{ik} , a_{ijkm} , ..., are parameters to be estimated for i, j = 1, ..., n; k, m = 1, ..., ∞ ; and v_i's are expenditure normalized prices. Also following Barnett and Yue (BY) (1988) we let the exponent set be $\Lambda = \{\lambda(k): \lambda(k) = 2^{-k}, k \in N\}$. BY shows that eliminating diagonal elements does not alter the properties of f(v) and so in what follows we consider f(v) for $i \neq j$. To fix ideas we believe it is instructive to present a simple form of the expansion, consequently with three goods and K = 1 ($\Rightarrow \Lambda = \{1/2\}$), that is, a first order expansion we have:

(2)
$$f_{k=1}(v) = a_0 + a_1 v_1^{1/2} + a_2 v_2^{1/2} + a_3 v_3^{1/2} + a_4 v_1^{1/2} v_2^{1/2} + a_5 v_1^{1/2} v_3^{1/2} + a_6 v_2^{1/2} v_3^{1/2} + a_7 v_1^{1/2} v_2^{1/2} v_3^{1/2}$$

where,
$$a_4 = a_{12} + a_{21}$$
, $a_5 = a_{13} + a_{31}$, $a_6 = a_{23} + a_{32}$, and
 $a_7 = a_{123} + a_{132} + a_{213} + a_{231} + a_{312} + a_{321}$.

Of course, the demand functions are found from the modified Roy's identity, hence if we denote demand for good i by q_i , then q_i is given by:

(3)
$$\mathbf{q}_{i} = \frac{\partial f(v) / \partial x_{i}}{\sum_{j=1}^{k} v_{j} \partial f(v) / \partial v_{j}}$$

Therefore the share equations are given by:

(4)
$$\mathbf{s}_{i} = \mathbf{v}_{i}\mathbf{q}_{i} = \frac{v_{i}\partial f(v)/\partial x_{i}}{\sum_{j=1}^{k} v_{j}\partial f(v)/\partial v_{j}}$$

Now, if we let

 $\begin{array}{l} \text{(5) } S_i = v_i \partial f(v) / \; \partial v_i \, , \; _{\forall \; i.} \\ \text{Then,} \\ \text{(6) } s_i = S_i / S \end{array}$

where,
$$\mathbf{S} = \sum_{\forall i} S_i$$

With this model setup in mind we want to specify explicitly the model to be estimated. In what follows we will estimate a system with three goods for K = 2. This renders that the equations we are interested in are:

$$(7) f_{k=2}(v) = b_{0} + b_{1} v_{1}^{1/2} + b_{2} v_{2}^{1/2} + b_{3} v_{3}^{1/2} + b_{4} v_{1}^{1/4} + b_{5} v_{2}^{1/4} + b_{6} v_{3}^{1/4} + b_{7} v_{1}^{1/2} v_{2}^{1/2} \\ + b_{8} v_{1}^{1/2} v_{2}^{1/4} + b_{9} v_{1}^{1/4} v_{2}^{1/2} + b_{10} v_{1}^{1/4} v_{2}^{1/4} + b_{11} v_{1}^{1/2} v_{3}^{1/2} + b_{12} v_{1}^{1/2} v_{3}^{1/4} \\ + b_{13} v_{1}^{1/4} v_{3}^{1/2} + b_{14} v_{1}^{1/4} v_{3}^{1/4} + b_{15} v_{2}^{1/2} v_{3}^{1/2} + b_{16} v_{2}^{1/2} v_{3}^{1/4} + b_{17} v_{2}^{1/4} v_{3}^{1/2} \\ + b_{18} v_{2}^{1/4} v_{3}^{1/4} + b_{19} v_{1}^{1/2} v_{2}^{1/2} v_{3}^{1/2} + b_{20} v_{1}^{1/4} v_{2}^{1/2} v_{3}^{1/2} + b_{21} v_{1}^{1/2} v_{2}^{1/4} v_{3}^{1/4} \\ + b_{22} v_{2}^{1/2} v_{2}^{1/2} v_{3}^{1/4} + b_{23} v_{1}^{1/2} v_{2}^{1/4} v_{3}^{1/4} + b_{24} v_{1}^{1/4} v_{2}^{1/2} v_{3}^{1/4} + b_{25} v_{1}^{1/4} v_{2}^{1/4} v_{3}^{1/2} \\ + b_{26} v_{1}^{1/4} v_{2}^{1/4} v_{3}^{1/4} v_{3}^{1/4} v_{3}^{1/4} + b_{23} v_{1}^{1/2} v_{2}^{1/4} v_{3}^{1/4} + b_{24} v_{1}^{1/4} v_{2}^{1/2} v_{3}^{1/4} + b_{25} v_{1}^{1/4} v_{2}^{1/4} v_{3}^{1/2} v_{3}^{1/4} \\ + b_{26} v_{1}^{1/4} v_{2}^{1/4} v_{3}^{1/4} v_{3}^{1$$

Since, we have only three goods, as is well known we need only estimate two of the share equations, which we can calculate from the following:

$$(8) 4S_{1} = 2b_{1} v_{1}^{1/2} + b_{4} v_{1}^{1/4} + 2b_{7} v_{1}^{1/2} v_{2}^{1/2} + 2b_{8} v_{1}^{1/2} v_{2}^{1/4} + b_{9} v_{1}^{1/4} v_{2}^{1/2} + b_{14} v_{1}^{1/2} v_{3}^{1/4} + 2b_{11} v_{1}^{1/2} v_{3}^{1/2} + 2b_{12} v_{1}^{1/2} v_{3}^{1/4} + b_{13} v_{1}^{1/4} v_{3}^{1/2} + b_{14} v_{1}^{1/4} v_{3}^{1/4} + 2b_{19} v_{1}^{1/2} v_{2}^{1/2} v_{3}^{1/2} + b_{20} v_{1}^{1/4} v_{2}^{1/2} v_{3}^{1/2} + 2b_{21} v_{1}^{1/2} v_{2}^{1/2} v_{3}^{1/4} + 2b_{22} v_{1}^{1/2} v_{2}^{1/2} v_{3}^{1/4} + 2b_{23} v_{1}^{1/2} v_{2}^{1/4} v_{3}^{1/4} + b_{24} v_{1}^{1/4} v_{2}^{1/2} v_{3}^{1/4} + b_{25} v_{1}^{1/4} v_{2}^{1/4} v_{3}^{1/4} + b_{26} v_{1}^{1/4} v_{2}^{1/4} v_{3}^{1/4} v_{3}^{1/4$$

$$(9) 4S_{2} = 2b_{2} v_{2}^{1/2} + b_{5} v_{2}^{1/4} + 2b_{7} v_{1}^{1/2} v_{2}^{1/2} + b_{8} v_{1}^{1/2} v_{2}^{1/4} + 2b_{9} v_{1}^{1/4} v_{2}^{1/2} + b_{10} v_{1}^{1/4} v_{2}^{1/4} + 2b_{15} v_{2}^{1/2} v_{3}^{1/2} + 2b_{16} v_{2}^{1/2} v_{3}^{1/4} + b_{17} v_{2}^{1/4} v_{3}^{1/2} + b_{18} v_{2}^{1/4} v_{3}^{1/4} + 2b_{19} v_{1}^{1/2} v_{2}^{1/2} v_{3}^{1/2} + 2b_{20} v_{1}^{1/4} v_{2}^{1/2} v_{3}^{1/2} + b_{21} v_{1}^{1/2} v_{2}^{1/4} v_{3}^{1/2} + 2b_{22} v_{2}^{1/2} v_{2}^{1/2} v_{3}^{1/4} + b_{23} v_{1}^{1/2} v_{2}^{1/4} v_{3}^{1/4} + 2b_{24} v_{1}^{1/4} v_{2}^{1/2} v_{3}^{1/4} + b_{25} v_{1}^{1/4} v_{2}^{1/4} v_{3}^{1/2} + b_{26} v_{1}^{1/4} v_{2}^{1/4} v_{3}^{1/4}$$

(10)
$$4\mathbf{S} = 2b_1 v_1^{1/2} + 2b_2 v_2^{1/2} + 2b_3 v_3^{1/2} + b_4 v_1^{1/4} + b_5 v_2^{1/4} + b_6 v_3^{1/4} + 4b_7 v_1^{1/2} v_2^{1/2}$$

$$+3b_{8} v_{1}^{1/2} v_{2}^{1/4} +3b_{9} v_{1}^{1/4} v_{2}^{1/2} +2b_{10} v_{1}^{1/4} v_{2}^{1/4} +4b_{11} v_{1}^{1/2} v_{3}^{1/2} +3b_{12} v_{1}^{1/2} v_{3}^{1/4} +3b_{13} v_{1}^{1/4} v_{3}^{1/2} +2b_{14} v_{1}^{1/4} v_{3}^{1/4} +4b_{15} v_{2}^{1/2} v_{3}^{1/2} +3b_{16} v_{2}^{1/2} v_{3}^{1/4} +3b_{17} v_{2}^{1/4} v_{3}^{1/4} +2b_{18} v_{2}^{1/4} v_{3}^{1/4} +6b_{19} v_{1}^{1/2} v_{2}^{1/2} v_{3}^{1/2} +5b_{20} v_{1}^{1/4} v_{2}^{1/2} v_{3}^{1/2} +5b_{21} v_{1}^{1/2} v_{2}^{1/4} v_{3}^{1/4} +5b_{22} v_{2}^{1/2} v_{2}^{1/2} v_{3}^{1/4} +4b_{23} v_{1}^{1/2} v_{2}^{1/4} v_{3}^{1/4} +4b_{24} v_{1}^{1/4} v_{2}^{1/2} v_{3}^{1/4} +4b_{25} v_{1}^{1/4} v_{2}^{1/4} v_{3}^{1/2} +3b_{26} v_{1}^{1/4} v_{2}^{1/4} v_{3}^{1/4} v_{3}^{1/4}$$

To impose the homogeneity property we impose the restriction $b_1 + b_2 + b_3 = 1$, hence we have one less parameter to estimate when we do the actual estimation. Also, the fact that we are using econometric techniques requires that we specify errors and how they enter the equations. Following the usual procedure the errors are allowed to enter the share equations additively implying that the actual equations estimated are:

$$\mathbf{s}_{i} = \mathbf{S}_{i} / \mathbf{S} + \mathbf{e}_{t}, \forall i.$$

DFS defines $S_i/S = h(v_t, \alpha)$

where α is the set of parameters to be estimated.

As mentioned earlier DFS considers two possibilities for capturing the dynamics of the money demand process; being an AR(1) representation of the errors or explicitly including the lagged shares in the estimated share equations. The AR(1) specification is especially suspect in this model because of the complex nature of the equations being estimated which results in common factor restrictions (CFR) that are almost intractable. As is well known CFR's result in biased estimates and since we cannot trace the CFRs, and therefore afford ourselves the possibility of testing them, this modeling strategy is not recommended. To make the argument explicit consider the simple model (Spanos 1986 Pp. 504 - 507):

(A)
$$y_t = \beta x_t + \xi_t$$

 $\xi_t = \rho \xi_{t-1} + u_t, 0 < \rho < 1, t \in T; u_t \sim \text{NIID} (0, \sigma^2).$

 $\zeta_t - p\zeta_{t-1} + u_t, 0$

(B)
$$y_t = \beta_0' x_t + \sum_{i=1}^m \alpha_i y_{t-i} + \sum_{i=1}^m \beta_i x_{t-i} + u_t, t > m; u_t \sim \text{NIID} (0, \sigma^2).$$

Note that model (A) implies that:

$$\xi_t = y_t - \beta x_t$$

Therefore solving recursively for ξ_{t-i} and substituting into y_t gives:

(C)
$$y_t = \beta' x_t + \sum_{i=1}^m \rho_i (y_{t-i} - \beta' x_{t-1}) + u_t$$

Now, comparing models (B) and (C) reveals that the two are only equivalent if the following is true:

 $\beta \rho_i = -\beta_i$, i = 1, 2, ..., m, which are in fact the implied common factor restrictions if one were to specify model (A), when the 'true' model is model (B).

Furthermore, DFS's argument for using the AR(1) specification is that the dynamic representation resulting from including the lagged shares as explanatory variables is not invariant to the omitted equation is a direct consequence of how they chose to represent the dynamics. In particular, they have a model with three goods and consider their specification:

$$(11) \ S_{it} = h_i(v_t, \, \alpha) + \delta_i h_i(v_{t\text{-}1}, \, \alpha) + \ b_{i1} S_{1t\text{-}1} + b_{21} S_{2t\text{-}1} + b_{31} S_{3t\text{-}1} + U_{it}$$

where, S_{it} is the actual share for good i in period t. However, the shares must sum to 1 in all periods and therefore including all the lagged shares on the right hand side of the equation results in a problem of multicollinearity, rendering that the parameter estimates will be very sensitive to the model specification. Therefore in specifying such a dynamic process it is judicious to drop one of the lagged shares from the list of explanatory variables. Thus, the model that will be estimated here, will have share equations of the form:

(12) $S_{it} = h_i(v_t, \alpha) + b_{i1}S_{1t-1} + b_{21}S_{2t-1} + U_{it}$, for i = 1, 2. 128

Section 3: The Data

The Data used here is a subset of a comprehensive data compilation referred to as the Monetary Services Index (MSI) at the Federal Reserve Bank of St. Louis. A detailed description of the data can be found in Anderson, Jones and Nesmith (AJN) (1997a, b, c). The MSI measures period by period flow of monetary services to households, deriving from their utilization of monetary assets. This renders that the prices that we refer to in this study are in fact user cost as defined by Donovan (1977) and formally derived by Barnett (1978). It is also worthwhile noting that monetary assets are in general not perfectly substitutable, and, thus, the index used is the TÖrnquist-Theil (the discrete time equivalent of the Divisia index), which in the monetary literature is simply referred to as the Divisia Monetary index.

To give a more concrete sense of the issues Barnett's formulae is:

$$p_{it} = p_{t}^* \frac{R_t - r_{it}}{1 + R_t},$$

where, p_{it} is the user cost of asset i in period t.

 p_{t}^{*} is an aggregate index of goods and services prices and of durable goods real

prices during period t.

 R_t is the yield on a bench mark asset.

 r_{it} is the nominal yield on asset i during period t.

Clearly, for the purposes of the model we intend to estimate we need income data which can be calculated as (see Anderson, Jones and Nesmith (1997c), table 1):

$$M_t = y_t = \sum_{i=1}^n p_{it}^{real} m_{it}^{nom}$$

where, y_t is total expenditure on monetary assets.

 m_{it}^{nom} is the nominal quantity of asset i in period t.

The Nominal TÖrnquist-Theil monetary services index (MSI) is then given by:

$$MSI_{t}^{nom} = MSI_{t-1}^{nom}\Pi_{i=1}^{n} \left[\frac{m_{it}^{nom}}{m_{it-1}^{nom}}\right]^{\overline{\varpi}}it$$

where, $\overline{\omega}_{it} = (w_{it} + w_{i,t-1})/2$, and w_{it} is simply the budget share of asset i in period t.

The data used is then, quarterly US data from 1960 to the first quarter of 2004. The data has been seasonally adjusted so that there is no need to include seasonal dummies into what is an already complex model. We assume that the monetary assets which comprise M2 are weakly separable from all other goods and therefore only these assets are included in the model we estimate. Following Yue (1991) the categories in which we group the monetary assets are:

- A1 = currency, demand deposits and other checkable deposits.
- A2 = Savings deposits in Commercial banks and thrifts, super NOW accounts and money
- market deposit accounts.
- A3 = small time deposits at commercial banks and thrifts.

Section 4: Estimation

The model given in (12) is estimated, but only for two of the share equations, and consistent with circumventing the collinearity problem the lagged share of A3 never appears in any of the estimated equations. The parameter estimates are given in table 1 and time series of the elasticities are reported¹ in table 2. These time series were generated by calculating the elasticity in each of the periods from 1974 fourth quarter, to the first quarter of 2004. One should note that for space constraints only some of the elasticities are reported, however, the remainder can be obtained from the author upon request. Due to the highly complex nature of the model we wish to specify the formulas for the elasticity estimates (see Yue (1991), p. 42 for the Allen-Partials; and DFS (2003), p. 110 for the Morishima's):

¹ All tables and graphs are reported in the appendix beginning at page 15.

<u>Compensated Allen Partials²</u>;

$$\sigma_{ij} = \frac{\partial A_i^c p_i}{\partial p_j A_i^c} = \left[\frac{1}{v_i} \frac{\partial s_i}{\partial p_i} + \frac{s_i}{v_j} \left(\frac{1}{v_i} \frac{\partial s_i}{\partial M} + \frac{s_i}{p_i}\right)\right] \frac{M v_i v_j}{s_i s_j}$$

for $i \neq j$, and (**NB**: all variables referred to here are consistent with previous definitions(see section 3))

$$\sigma_{ij} = \frac{\partial A_i^c p_i}{\partial p_j A_i^c} = \left[\frac{1}{v_i} \frac{\partial s_i}{\partial p_i} - \frac{s_i}{v_i p_i} + \frac{s_i}{v_i} \left(\frac{1}{v_i} \frac{\partial s_i}{\partial M} + \frac{s_i}{p_i}\right)\right] \frac{M v_i^2}{s_i^2}$$

for $i \neq j$.

Income Elasticities:

$$\eta_{i0} = \frac{\partial A_i M}{\partial M A_i} = \frac{\partial s_i}{\partial M} \frac{M}{s_i} + 1$$

Thus, the uncompensated Allen Partials are:

$$\eta_{ij} = \frac{\partial A_i p_i}{\partial p_j A_i} = s_j \sigma_{ij} - s_j \eta_{io}$$

Therefore, the Morishima elasticities are:

 $\eta_{ii}^{M} = s_{i}(\eta_{ii} - \eta_{ii})$, note that all own elasticities here are zero.

As Blackorby and Russell make abundantly clear, the Allen Partials are inappropriate due to their symmetry as well as the fact that they may indicate that assets are substitutes when they are in fact complements; we regard the Allen Partials as unreliable. Contingent on this line of reasoning the discussion that follows focuses on the Morishima elasticities, even though, the Allen partials are still reported for completeness. Of course, it should be noted that the Morishima elasticities presents a troubling inconsistency in that it may suggests that good i is a complement to good j, but that good j is a substitute to good i. Again, the reader is reminded that the elasticity estimates presented are for the fourth quarter of 1974 through to the first quarter of 2004, hence the reference 1, 2 and 7 in the quarters section in the table and plots refers to, respectively, the fourth quarter of 1974; the first quarter of 1975 and the second quarter of 1976. The remainder of the estimates should be interpreted in similar fashion.

Do our Empirical Findings concur with the theory?

Since, the own Morishima elasticities are zero by construction the metric available to us as a check on the law of demand is the own Allen-Partials. As figure 1 shows all these estimates are negative for A1 (similar results obtain for A2 and A3), hence we can conclude that the law of demand is satisfied empirically. Figures 4-7, with all the elasticity estimates positive, confirms that A1, A2, and A3 are substitutes, thus confirming the *a*-*priori* expectation arising from the theory. However, there still remains one gray area represented in the fact that figure 8 suggests that A3 and A2 are complements, while, on the other hand, figure 9 suggests that A2 and A3, for most of the sample period are substitutes. This is exactly the inconsistency, which may arise from using Morishima elasticities, in that there is no guarantee that the cross elasticities between any two goods will have the same sign for cross elasticity ij versus ji. Naturally, in any model of money demand one would want to test the long standing hypotheses surrounding money demand. For instance, money neutrality is a key theoretical issue and should be tested. Unfortunately, no measure of the overall price level³ appears explicitly in this model and so that postulate is not testable in the framework.

 $^{^{2}}$ The model estimation was conducted using the SAS system 8.0, however, the derivatives that appear in the elasticity estimates were obtained using Mathematica 5.0.

³ Had we not postulated that monetary assets are weakly separable from all other goods, then an all other goods "asset" would appear in the model, and it's user cost would have been some measure of the overall price index, from which we could test the money neutrality hypothesis.

Another of the major hypotheses is the unitary income elasticity of money demand (see, among other studies, Yue (1991), p.45), and in this empirical exercise it is rather interesting that both A2, which corresponds to Poole's definition of MZM (see AJN (1997c)) and A3, which is essentially M2, both have income elasticities equal to one. Verification of this result is presented in Table 1 and figures 11 and 12. The result is rather robust, as it obtains for all periods including the anomalous periods from the mid to late 1970's into the early 1980's (see, *inter alia*, Yue (1991), p.46). In contrast to these it seems that M1, which is identical to A1 here, is too narrow a measure of money to adequately reflect its properties. In fact, the introduction of super NOW accounts and other monetary assets which not only almost fully replicate the liquidity properties of currency, but also have interest bearing characteristics, seems to have resulted in a complete reversal of the income elasticity of A1 in the late 1970's (see figure 10). That is, agents in the economy no longer demanded more notes and coins in response to increases in income, but quite the contrary, chose to hold less currency in response to changes in income.

Furthermore, the discussion on money demand sought to address the issue of whether or not the money demand process is stable. This study sheds some light on the issue, but does not afford us any unequivocal conclusions. In particular, the stability of the of the income elasticity estimates suggests that the money demand function is stable. Contrary to this though, we find that the cross elasticities range in some instances from 0.1 to 0.7, which represent up to a 600 per cent change in the elasticity estimates over the period. This would suggest that the elasticity estimates are highly unstable. Although, one should bear in mind the substantial innovations in the market for monetary assets over the period, which caused significant changes in user cost and asset shares of total expenditure on monetary assets.

Section 5: Conclusion

Microeconomic estimation of demand systems has matured to the stage that they now rival crude aggregated structural models of economic activity. The asymptotically ideal model, first semiparametrically estimated by Barnett and Yue (1988) is used here to produce estimation results generally consistent with the theory underlying the money demand process. The major findings of the discourse are that the unitary income elasticity hypothesis is satisfied, and in general monetary assets are substitutes as well as the law of demand is satisfied by the model. However, no definitive answers could be found as to whether or not the money demand process is stable, but this may be due to the rapid rate of innovation in this market. Further research should test whether or not monetary assets are weakly separable from all other goods, and if not allow for the testing of the money neutrality hypothesis. Of importance for future considerations are also obtaining a stable money demand function. It is important that the trend of doing microeconometric estimation of such systems is continued, as it seems rather promising. In light of this, we need be reminded that the user cost of monetary assets is the appropriate price. Also, in forming monetary aggregates it is desirable to use Divisia indices which allow for different degrees of substitutability of the assets, as opposed to simple sum indices, which implicitly assumes that all assets are perfect substitutes.

References

Anderson, R. G., Jones, B. E. and Nesmith, T. D. (1997). Introduction to the St. Louis Monetary Services Index. *Federal Reserve Bank of St. Louis Economic Review*, **79**, 25-29.

----- and ----- (1997). Monetary aggregation theory and statistical index numbers. *Federal Reserve Bank of St. Louis Economic Review*, **79**, 31-51.

----- and ----- (1997). Building Monetary Services Indexes: concepts, data and methods . *Federal Reserve Bank of St. Louis Economic Review*, **79**, 53-82.

Baba, Y., Hendry, D. F. and Starr, R. M. (1992). The demand for M1 in the USA 1960-1988. *Review of Economic Studies*, **59**, 25-61.

Barnett, W.A. (1978). The user cost of money. *Economic Letters*, 1, 145-9.

----- (1979). Theoretical Foundations for the Rotterdam Model. Review of Economic Studies, 46, 109-30.

----- (1983). The flexible Laurent demand system. *Proceedings of the 1982 American Statistical Association Meetings*.

-----, Geweke, J. and Yue, P. (1991). Seminonparametric Bayesian estimation of the Asymptotically Ideal Model: the AIM Consumer Demand System. In W. Barnett, J. Powell and G. Tauchen (eds.), Nonparametric and Seminonparametric Methods in Econometrics and Statistics, *Proceedings of the Fifth International Symposium in Economic Theory and Econometrics* (Cambridge University Press, 1991).

----- and Jonas, A. (1983). The Muntz-Szatz demand system: an application of a globally well behaved series expansion. *Economic Letters*, **11**, 337-42.

----- and Yue, P. (1988). Seminonparametric estimation of the Asymptotically Ideal Model: the AIM Demand System. In G. Rhodes and T. Fomby (eds.), Nonparametric and Robust Inference, *Advances in Econometrics*, 7, 229-51.

Barr, D. G. and Cuthbertson, K. (1991). Neoclassical consumer demand theory and the demand for money. *Economic Journal*, **101**, 855-76.

Bashier, A. and Dahlan, A. (2011). The Money Demand Function for Jordan: An Empirical Investigation. *International Journal of Business and Social Science*, **2**, **5**, [Special Issue -March 2011], 77-86.

Blackorby, C. and Russell, R. R. (1981). The Morishima elasticity of substitution: symmetry, constancy, separability and its relationship to the Hicks and Allen elasticities. *Review of Economic Studies*, **48**, 147-58.

----- and ----- (1989). Will the real elasticity of substitution please stand up? *American Economic Review*, **79**, 882-8.

Christensen, L. R., Jorgensen, D. W. and Lau, L. J. (1975). Transcendental Logarithmic Utility Functions. *American Economic Review*, **65**, 367-83.

Deaton, A. S. and Muellbauer, J. (1980). An Almost Ideal Demand System. *American Economic Review*, **70**, 312-26.

Diewert, W. E. and Wales, T. J. (1987). Flexible functional forms and global curvature conditions. *Econometrica*, **55**, 43-68.

Fisher D. (1992). Money-demand variability: a demand-systems approach. *Journal of Business Economics and Statistics*, **10**, 143-51.

----- and Fleissig, A. R. (1994) Money Demand in a dynamic Fourier expenditure system. *Federal Reserve Bank of St. Louis Economic Review*, **76**, 117-28.

----- and ----- (1997). Monetary aggregation and the demand for monetary assets. *Journal of Money Credit* and Banking, **29**, pt 1, 458-75.

Fleissig, A. R. and Swofford, J. L. (1997) Dynamic asymptotically ideal models and finite approximation. *Journal of Business Economics and Statistics*, **15**, 482-92.

Friedman, M. (1956). The quantity theory of money: a restatement. In M. Friedman (ed.), *Studies in the Quantity Theory of Money*. Chicago: Chicago University Press.

Gallant, A. R. (1981). On the bias in flexible functional forms and an essentially unbiased form: the Fourier flexible form. *Journal of Econometrics*, **15**, 211-45.

Hendry, D. F. and Mizon, G. E. (1978). Serial correlation as a convenient simplification, not a nuisance: a comment on the study of the demand for money by the Bank of England. *Economic Journal*, **88**, 549-63.

Michael, P., Nobay, A. R., and Peel, D. A. (1994). German Hyperinflation and the Demand for money revisited. *International Economic Review*, **35**, 1-22.

Narayan, P., (2009). Estimating money demand_functions for South Asian countries. *Empirical Economics*, **36**, 3, 685-696.

Samreth, S (2008): *Estimating Money Demand Function in Cambodia: ARDL Approach*. Unpublished. Retrieved from: http://mpra.ub.uni-muenchen.de/16274/.

Spanos, A. (1986). Statistical foundations of econometric modelling: Cambridge University.

Swofford, J. and Whitney, G. (1987). Nonparametric tests of utility maximization and weak separability for consumption, leisure and money. *Review of Economics and Statistics*, **69**, 458-64.

----- and ----- (1988). A comparison of non-parametric tests of utility maximization and weak separability for annual and quarterly data on consumption, leisure, and money. *Journal of Business and Economics Statistics*, **6**, 241-6.

Varian, H. R. (1982). The nonparametric approach to demand analysis. *Econometrica*, **50**, 945-73.

Varian, H. R. (1983). Non-parametric tests of consumer choice. The Review of Economic Studies, 50, 99-110.

Wales, T. J. (1977). On the flexibility of flexible functional forms: an empirical approach. *Journal of Econometrics*, **5**, 183-193.

White, H. (1980). Using Least squares to approximate unknown regression functions. *International Economic Review*, **21**, 149-70.

Yue, P. (1991). A Microeconometric Approach to Estimating Money Demand: the Asymptotically Ideal Model. *Federal Reserve Bank of St. Louis Economic Review*, **73**, 36-51.

Appendix: Tables and Graphs

Table 1: ITSUR Parameter Estimates

Ν	onlinear ITSU	R Summary	of Resid	ual Errors				
DF	DF		Adj					
Moc	lel Error	SSE I	MSE Ro	oot MSE F	R-Square	R-Sq		
125	1625 0.0	246 0.000	212 0	0146 0.0	955 0.00	4.4		
13.5	162.5 0.0	<u>540 0.000</u>	$\frac{213}{228}$ 0.	0140 0.9	$\frac{833}{469}$ 0.98	27		
15.5	102.3 0.0.	549 0.000.	558 0.	0184 0.9	408 0.94	-27		
	Nonlinear ITS	SUR Parame	ter Estim	ates				
			Lot Lotin	aces				
	Ap	prox	Appi	OX				
Parameter Estimate Std Err t Value $Pr > t $								
k1	0.001133	0.000240	4.73	<.0001				
k2	0.000847	0.000221	3.83	0.0002				
b1	0.396073	0.0559	7.09	<.0001				
b2	0.632676	0.0586	10.79	<.0001				
<u>b4</u>	0.033606	0.0279	1.20	0.2309				
b5	0.05126	0.0273	1.88	0.0622				
b6	0.165255	0.0710	2.33	0.0211				
b/	0.303556	0.0856	3.55	0.0005				
b8 b0	0.041539	0.0680	0.61	0.5419				
b9 b10	-0.4/1/0	0.2034	-2.32	0.0210				
b10	-0.91339	0.1735	-5.21	<.0001				
b12	1.014202	0.0973	11.40	< 0001				
b13	-0.23702	0.0737	-1.45	0.1485				
b13	-0.17707	0.1289	-1.45	0.1715				
b15	0.68555	0.0693	9.89	<.0001				
b16	-1.15654	0.0977	-11.83	<.0001				
b17	0.337248	0.1952	1.73	0.0859				
b18	-0.53923	0.1341	-4.02	<.0001				
b19	0.009497	0.0972	0.10	0.9223				
b20	0.07711	0.1775	0.43	0.6645				
b21	0.022534	0.1454	0.15	0.8770				
b22	-0.00657	0.1397	-0.05	0.9626				
b23	-0.65807	0.1625	-4.05	<.0001				
b24	0.043917	0.2759	0.16	0.8737				
b25	-1.83962	0.3746	-4.91	<.0001				
b26	3.295423	0.3436	9.59	<.0001				
	N DF Moo 13.5 13.5 13.5 13.5 13.5 13.5 13.5 13 5 10 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	Nonlinear ITSU DF DF Model Error 13.5 162.5 0.0 13.5 162.5 0.0 13.5 162.5 0.0 13.5 162.5 0.0 13.5 162.5 0.0 13.5 162.5 0.0 Nonlinear ITS Ag Parameter Estima k1 0.001133 k2 0.000847 b1 0.396073 b2 0.632676 b4 0.033606 b5 0.05126 b6 0.165255 b7 0.303556 b8 0.041539 b9 -0.47176 b10 -0.91359 b11 1.014202 b12 -1.03568 b13 -0.23702 b14 -0.17707 b15 0.68555 b16 -1.15654 b17	Nonlinear ITSUR Summary DF SE N Model Error SSE N 13.5 162.5 0.0346 0.0007 13.5 162.5 0.0549 0.0007 13.5 162.5 0.0549 0.0007 Image: State Stat	Nonlinear ITSUR Summary of Resident Summary of Res	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Nonlinear ITSUR Summary of Residual Errors Adj Adj Model Error SSE MSE Rodi MSE R-Square 13.5 162.5 0.0346 0.000213 0.0146 0.9855 0.98 13.5 162.5 0.0549 0.000338 0.0184 0.9468 0.94 Nonlinear ITSUR Parameter Estimates Approx Parameter Std Err t Value Pr > t k1 0.001133 0.000240 4.73 <.0001 k1 0.001133 0.00021 3.83 0.0001 b1 0.00021 3.83 0.0001 b2 0.63267 0.00559 7.09 <.0001 b1 0.10525 0.1079 <.0001 b	Nonlinear ITSUR Summary of Residual Errors Adj Model Error SSE MSE R-Square R-Sq 13.5 162.5 0.0346 0.000213 0.0146 0.9855 0.9844 13.5 162.5 0.0549 0.000338 0.0184 0.9468 0.9427 Nonlinear ITSUR Parameter Estimates Parameter Estimate Approx Approx Prevint Value Pr > t k1 0.001133 0.00021 3.83 0.0002 b1 0.396073 0.0559 7.09 L 0.001133 0.0002 b1 0.396073 0.0529 L 0.0011 b2 0.632676 0.05360 0.02134	

Table 2: Morishima and Income Elasticities.

quarter	M12	M21	M13	M31	M23	M32	Y10	Y20	Y30
1	0.0779	0.049	0.0931	0.0343	0.0202	-8.5E-12	0.5096	1	1
2	0.078	0.0505	0.0887	0.039	0.0147	-1E-11	0.4827	1	1
3	0.0724	0.0556	0.076	0.0507	0.0054	-1.3E-11	0.4217	1	1
4	0.069	0.0597	0.0704	0.0576	0.0023	-1.5E-11	0.3838	1	1
5	0.0707	0.0612	0.0729	0.0577	0.0037	-1.5E-11	0.3728	1	1
6	0.0707	0.0599	0.0738	0.0552	0.0051	-1.5E-11	0.3879	1	1
7	0.069	0.058	0.0719	0.0539	0.0045	-1.4E-11	0.4051	1	1
8	0.0693	0.0552	0.075	0.0478	0.0086	-1.3E-11	0.4404	1	1
9	0.0749	0.0497	0.0854	0.0387	0.0143	-1.1E-11	0.4931	1	1
10	0.072	0.0515	0.0771	0.0454	0.0073	-1.3E-11	0.463	1	1
11	0.0725	0.0509	0.076	0.0466	0.005	-1.4E-11	0.4608	1	1
12	0.0725	0.0492	0.0761	0.045	0.0051	-1.3E-11	0.4754	1	1
13	0.0706	0.0532	0.0693	0.055	-0.002	-1.6E-11	0.4216	1	1
14	0.0698	0.0596	0.0682	0.0622	-0.003	-1.8E-11	0.3665	1	1
15	0.0712	0.0658	0.066	0.0769	-0.01	-2.2E-11	0.2842	1	1
16	0.0724	0.0645	0.0719	0.0653	-8E-04	-1.8E-11	0.3274	1	1
17	0.0844	0.0903	0.0853	0.0873	0.0022	-2.3E-11	0.1111	1	1
18	0.0843	0.0817	0.0923	0.0628	0.0168	-1.6E-11	0.232	1	1
19	0.08/3	0.0732	0.0961	0.056	0.0163	-1.5E-11	0.2885	1	1
20	0.0999	0.0815	0.0975	0.0897	-0.006	-2./E-11	0.0997	1	1
21	0.1242	0.1195	0.1221 0.1342	0.1022	-0.023	-3.4E-11 5.4E-11	-0.451	1	1
22	0.1348	0.1299	0.1342 0.127	0.1850	-0.021	-8.9E-11	-0.509	1	1
23	0.1271	0.0224	0.127	0.253	-0.049	-8 3E-11	-0.56	1	1
25	0.1534	0.1487	0.1591	0.2525	-0.031	-7.6E-11	-0.861	1	1
26	0.1525	0.1223	0.156	0.2246	-0.033	-7.6E-11	-0.683	1	1
27	0.1593	0.1286	0.1642	0.2278	-0.031	-7.7E-11	-0.74	1	1
28	0.1595	0.1245	0.1739	0.3154	-0.047	-1.1E-10	-1.014	1	1
29	0.1451	0.1109	0.1662	0.4018	-0.06	-1.5E-10	-1.189	1	1
30	0.1534	0.1245	0.1737	0.3792	-0.056	-1.3E-10	-1.192	1	1
31	0.1514	0.1148	0.1716	0.3778	-0.057	-1.4E-10	-1.151	1	1
32	0.1529	0.1272	0.2058	0.6526	-0.072	-2.3E-10	-2.07	1	1
33	0.1465	0.1227	0.1792	0.5038	-0.066	-1.8E-10	-1.553	1	1
34	0.1555	0.1775	0.1691	0.3546	-0.043	-9.8E-11	-1.269	1	1
35	0.1483	0.1729	0.1539	0.2722	-0.03	-7.3E-11	-0.974	1	1
36	0.1499	0.1712	0.1557	0.2723	-0.03	-7.4E-11	-0.975	1	1
37	0.1484	0.1533	0.1503	0.2092	-0.02	-6E-11	-0.718	1	1
38	0.1515	0.1539	0.1536	0.2075	-0.019	-6E-11	-0.726	1	1
39	0.1671	0.1769	0.1776	0.3012	-0.033	-8.7E-11	-1.153	1	1
40	0.163	0.1508	0.1676	0.2269	-0.024	-7.1E-11	-0.823	1	1
41	0.1543	0.1393	0.1614	0.2638	-0.036	-8.5E-11	-0.872	1	1
42	0.1647	0.1816	0.1767	0.3244	-0.036	-9.2E-11	-1.227	1	1
43	0 1665	0 1855	0 1779	0.3184	-0.034	-8 9E-11	-1 228	1	1
44	0 1665	0 1907	0 1766	0 3103	-0.031	-8 5E-11	-1 217	1	1
45	0.1593	0.1697	0.1646	0.2506	-0.025	-7.2E-11	-0.942	1	1
46	0 1437	0 1332	0 1436	0 1698	-0.015	-5 2E-11	-0 507	1	1
47	0.1402	0.1306	0.1403	0 1901	-0.022	-5 9E-11	-0.552	1	1
48	0.15	0.1576	0.1533	0.1201	-0.024	-6 7E-11	-0.808	1	1
49	0.1575	0.1766	0.1607	0.2313	-0.018	-6 4E-11	-0 894	1	1
50	0.1575	0.1700	0.151	0.1887	-0.01	-5 4E-11	-0.681	1	1
51	0.1501	0.1003	0.175	0.1007	-0.022	-7 4E-11	-1 128	1	1
52	0 1700	0.2152	0 18/0	0.2727	-0.015	_7 1E_11	_1 208	1	1
52 53	0.1777	0.2132	0.1049	0.2070	-0.015	-7.112-11	-1.200	1	1 1
54	0.17944	0.2209	0.2137	0.3034	-0.030	-1.1E-10 _8 1E 11	-1.05	1 1	1 1
5 1 55	0.1703	0.1711	0.10/1	0.2000	-0.024	-0.1E-11	-1.170	1 1	1 1
55 56	0.1000	0.2033	0.1902	0.2950	-0.023	-0.41-11 _77E 11	-1.270	1	1 1
57	0.1673	0.1074	0.1921	0.2017	-0.02	-7.72-11	-1.137	1	1 1
51	0.10/5	0.140/	0.1/02	0.1273	-0.010	-0.56-11	-0.134	1	T

58	0.1663	0.1245	0.1665	0.1464	-0.009	-5.1E-11	-0.472	1	1
59	0.1737	0.0891	0.1862	0.0461	0.0323	-1.7E-11	0.0669	1	1
60	0.1691	0.0799	0.1741	0.0611	0.0127	-2.5E-11	0.0499	1	1
61	0.1642	0.0881	0.1651	0.0825	0.0032	-3.3E-11	-0.071	1	1
62	0.1635	0.1143	0.1632	0.1289	-0.006	-4.7E-11	-0.361	1	1
63	0.1683	0.1267	0.169	0.1555	-0.011	-5.5E-11	-0.519	1	1
64	0.174	0.1354	0.1743	0.1471	-0.005	-5E-11	-0.539	1	1
65	0.1824	0.1516	0.1876	0.2135	-0.019	-7.2E-11	-0.855	1	1
66	0.1828	0.1681	0.1948	0.285	-0.031	-9.2E-11	-1.142	1	1
67	0.1862	0.1927	0.2012	0.3285	-0.033	-9.8E-11	-1.363	1	1
68	0.1869	0.2005	0.1969	0.2938	-0.024	-8.5E-11	-1.279	1	1
69	0.1965	0.2199	0.2078	0.3154	-0.023	-8.9E-11	-1.439	1	1
70	0.2239	0.2931	0.2341	0.3677	-0.016	-9.1E-11	-1.905	1	1
71	0.2326	0.3243	0.2332	0.3283	-9E-04	-7.7E-11	-1.899	1	1
72	0.2336	0.3267	0.2275	0.2836	0.0108	-6.6E-11	-1.771	1	1
73	0.2552	0.3638	0.2481	0.3177	0.0105	-7.2E-11	-2.06	1	1
74	0.2383	0.3305	0.2259	0.2463	0.0226	-5.7E-11	-1.682	1	1
75	0.2347	0.3199	0.2196	0.2164	0.0298	-5E-11	-1.544	1	1
76	0.2193	0.2862	0.2054	0.181	0.0341	-4.3E-11	-1.278	1	1
77	0.2219	0.2822	0.2075	0.1748	0.0352	-4.2E-11	-1.256	1	1
78	0.2368	0.306	0.2202	0.1952	0.0334	-4.7E-11	-1.445	1	1
79	0.2633	0.3451	0.2439	0.2322	0.0298	-5.5E-11	-1.773	1	1
80	0.2638	0.3252	0.2424	0.2045	0.0337	-5E-11	-1.63	1	1
81	0.2677	0.3097	0.2493	0.2099	0.0272	-5.4E-11	-1.623	1	1
82	0.2434	0.242	0.2264	0.135	0.0377	-3.8E-11	-1.075	1	1
83	0.2229	0.1897	0.2127	0.0798	0.0515	-2.3E-11	-0.621	1	1
84	0.22	0.1955	0.2097	0.0939	0.0447	-2.8E-11	-0.691	1	1
85	0.207	0.182	0.2009	0.0745	0.0541	-2.1E-11	-0.524	1	1
86	0.2115	0 2 1 4 3	0 2003	0 1027	0.0479	-2.8E-11	-0.76	1	1
87	0.2364	0.2859	0.2177	0.1627	0.0405	-4 1F-11	-1 277	1	1
88	0.2304	0.2037	0.2177	0.1521	0.0405	-3 6E-11	-1 276	1	1
89	0.2317	0.2904	0.2127	0.1243	0.0500	-2 8F-11	-1.093	1	1
90	0.217	0.2204	0.1272	0.1245	0.0657	-3E-11	-1 258	1	1
91	0.2273	0.3463	0.2021	0.1609	0.0614	-3 4E-11	-1 428	1	1
92	0.2143	0 3139	0.1901	0 1138	0.0801	-2.4E-11	-1 106	1	1
93	0.203	0.2997	0.1819	0.096	0.0895	-2E-11	-0.965	1	1
94	0.197	0.2992	0.1766	0.0747	0.1092	-1.4E-11	-0.864	1	1
95	0.2046	0.3266	0.1795	0.0949	0.101	-1.8E-11	-1.043	1	1
96	0.2006	0.3269	0.1756	0.0794	0.1157	-1.5E-11	-0.975	1	1
97	0.2281	0.3809	0.1927	0.1207	0.0975	-2.3E-11	-1.363	1	1
98	0.2398	0.4444	0.1925	0.1165	0.1201	-2E-11	-1.562	1	1
99	0.2703	0.5336	0.211	0.158	0.1152	-2.5E-11	-2.043	1	1
100	0.2929	0.6063	0.2202	0.1667	0.1262	-2.5E-11	-2.342	1	1
101	0.3029	0.629	0.2245	0.1673	0.1299	-2.5E-11	-2.438	1	1
102	0.3058	0.6562	0.2205	0.1555	0.1433	-2.2E-11	-2.481	1	1
103	0.3202	0.7081	0.2222	0.149	0.1577	-2E-11	-2.646	1	1
104	0.2845	0.6498	0.1873	0.0803	0.2048	-8.8E-12	-2.14	1	1
105	0.277	0.646	0.1794	0.067	0.2205	-6.5E-12	-2.059	1	1
106	0.2859	0.6618	0.1917	0.0961	0.1933	-1.2E-11	-2.234	1	1
107	0.3452	0.8127	0.2441	0.2107	0.144	-2.8E-11	-3.206	1	1
108	0.3576	0.8489	0.2463	0.2027	0.1536	-2.7E-11	-3.32	1	1
109	0.4056	1.0159	0.2896	0.3092	0.1339	-4E-11	-4.252	1	1
110	0.4317	1.1397	0.3024	0.3354	0.1423	-4.1E-11	-4.748	1	1
111	0.442	1.2119	0.298	0.3155	0.1598	-3.6E-11	-4.92	1	1
112	0.4168	1.1596	0.2696	0.2514	0.1811	-2.8E-11	-4.508	1	1
113	0.4392	1.2566	0.2809	0.271	0.1853	-2.9E-11	-4.906	1	1
114	0.4183	1.221	0.2661	0.2515	0.1919	-2.7E-11	-4.686	1	1
115	0.382	1.1338	0.2429	0.2201	0.1988	-2.3E-11	-4.238	1	1
116	0 4292	1 3310	0.2641	0 249	0 2101	_2.5E 11	-5.03	1	1
117	0.4292	1 30/7	0.2041	0.279	0 2170	_2.5E-11	_1 86	1	1
118	0.7109	1 2202	0.2333	0.2200	0.2179	-2.26-11 -1 0F 11	_1 /26	1	1
110	0.30/7	1.4403	0.2520	0.174J	0.2210	-1.76-11	-+.+.0	1	1

Figure 1: Own Allen Partial for A1



Figure 2: Allen Partial between A1 and A2



Figure 3: Allen Partial between A1 and A3



Figure 4: Morishima Partial between A1 and A2











Figure 7: Morishima Partial between A3 and A1



Figure 8: Morishima elasticity between A2 and A3



Figure 9: Morishima elasticity between A3 and A2



Figure 10: Income elasticity of A1



Figure 11: Income elasticity of A2



Figure 12: Income elasticity of A3

