

## DISTRIBUTIVE EFFICIENCY IN JURISPRUDENCE

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### ABSTRACT

*This paper derives efficiency criteria that can be applied by a judge who must render a judgment vesting a right. Two litigants each claim a right to assign a numerical value to something. The judge has the jurisdictional power to render a declaratory judgment vesting the right exclusively in either party. Or he may exercise his discretion by assigning the numerical value to a party. The analysis in this paper is based on an assumption that the jurisprudence requires maximization of a social welfare function, defined as the sum of the litigants' utility functions. The results compare the distributive efficiency of a declaratory judgment with the distributive efficiency of a discretionary judgment. The results establish a decision criterion that is distributively efficient in the sense that it maximizes the social welfare function when the judge is imperfectly informed as to the litigant's valuations.*

**KEYWORDS:** DECLARATORY JUDGEMENT, ASYMMETRIC INFORMATION, SIGNALING, DISTRIBUTIONAL EFFICIENCY

### 1 THE NATURE OF THE LEGAL CONFLICT

Consider a case where there are two parties in litigation; call them Party 1 and Party 2. The only issue to be resolved by the litigation is the right to assign a numerical value to an entitlement<sup>1</sup>. Its numerical value is symbolized by  $X$ . The numerical value of  $X$  is assumed to be continuous within some well-defined range. Each litigant is assumed to have a unique optimal value of  $X$ . It is the undisclosed value to each litigant of having an exclusive right to the entitlement. If Party 1 is adjudged to have the exclusive right, I assume he will exercise his right by assigning to  $X$  the numerical value corresponding to his own optimum. The optimum for Party 1 is symbolized by  $X_1$ . Likewise if Party 2 is adjudged to have the exclusive right, I assume he will exercise the right by assigning to  $X$  his own optimal value, symbolized by  $X_2$ . I (arbitrarily) assume that the optimal value for Party 1 is less than that for Party 2, i.e.  $X_1 < X_2$ .

### 2 THE DEFINITION AND SCOPE OF JUDICIAL POWERS

The litigation is conducted as a bench trial in which the judge will render a declaratory judgment. A *declaratory judgment* is a judgment of a court which determines the rights of parties without ordering anything be done or awarding damages. In the case considered in this paper, the judgment determines that the contested right is vested exclusively in one of the parties. The form of vesting is assumed to be a judicial assignment of a property right.<sup>2</sup> In the context of this case, a *discretionary judgment* means the court has the lawful power to decide the case before him based upon a consideration of all factors involved as opposed to have to decide based upon a predefined legal guideline or rule. I assume that the jurisdictional power of the judge permits, but does not require, him to exercise his judicial discretion by assigning a numerical value of  $X$ .<sup>3</sup> The judge is not legally constrained as to the numerical value he may determine in the exercise of his discretion, except that it must lie within the well-defined range identified by the evidence adduced at trial.

### 3 COMMON KNOWLEDGE AND INFORMATION ASYMMETRY

Some information is common knowledge. The parties and the judge know that  $X_1 < X_2$ .

<sup>1</sup> I am using the word "entitlement" to mean the same thing as Ayers [2, pp. 4-5] "The notion of a legal 'entitlement' is an expansive one, encompassing such diverse rights as the right to bodily security, the right to a pollution-free atmosphere, the right to build a house that blocks another's view, and the right to damage another's reputation by false accusation."

<sup>2</sup> In Bebchuk's article [3, p. 606] he analyses the differing incentives and the likely economic consequences of property rules and liability rules as legal means for allocating legal rights. He states: "This analysis will show that from the perspective of ex ante efficiency, liability rules are not generally superior to property rights."

<sup>3</sup> The power of the trial judge to assign a numerical value to the entitlement may be conferred by a stipulation entered into by the parties. In some jurisdictions that judicial power is inherent in the common law doctrines of equity.

Beyond this common knowledge, information is distributed asymmetrically. In the general case, I assume that the optimal value of each party is undisclosed to his adversary and is not known by the judge. Each party believes it to be in his self-interest to keep that information strictly private. Private information is a transaction cost, especially in an adversarial forum where, for procedural or other reasons, the parties' negotiations are usually conducted through agents. In such contexts, the self-interest of the parties may induce each of them to strategically misrepresent their private valuations. Moreover, each party recognizes that his adversary has the same incentive as himself to misrepresent.<sup>4</sup> The asymmetric distribution of information affects judicial decision-making as well. Because the information available to the judge is incomplete, he will regard the optimal value for each party as a random variable. Prior to receiving evidence, the judge forms an *a priori* joint probability distribution governing the optimal assignments of the parties. The distribution is characterized by estimable parameters consisting of expected values, variances and the covariance.

The judge weighs the evidence adduced at trial and uses that evidence to adjust the parameters of a posterior probability distribution. The probability calculations may be carried out by the judge more or less subconsciously. The model is general enough to represent characteristics of many kinds of litigation: e.g. a conflict over partition of real property and/or the right to apportion property maintenance expenses, child custody litigation, environmental litigation fixing the assignment of pollution rights and fees, challenges to condemnation proceedings of real property pursuant to an eminent domain taking, ownership of a family business, a divorce, a declaration that a parcel of land is or is not zoned for commercial use, etc. Some of these disputes are amenable to a monetary expression or have monetary implications. Others are indivisible, e.g. there is no recognized legal right to a partial divorce. For additional examples, see Shavell [36].

#### 4 THE UTILITY FUNCTIONS OF THE LITIGANTS

The definition and analysis of distributive efficiency can be explored by applying a utilitarian approach. I assume that the utility function for each party has the property that as the difference between his optimal value of  $X$  and its adjudicated value increases, his post-verdict utility diminishes monotonically. I also assume that for both parties, any deviation from their respective optima will result in a loss of utility that increases geometrically as the deviation increases. The utility functions of the litigants are assumed to be quadratic expressions of the form<sup>5</sup>:

$$U_i(X) = -\theta_i(X - X_i)^2 \quad (1)$$

The symbols  $\theta_1$  and  $\theta_2$  represent the utility weights assigned to the outcome by each party. They are both positive numbers. These weights can be construed as the *stakes* of the litigation; the numerical value of each parameter represents the subjective importance each party assigns to the difference between his optimal outcome and the adjudicated outcome. One would expect that as a party's stake increases, *ceteris paribus*, that party would be willing to incur a greater cost (manifested in pecuniary terms or in effort) of transmitting to the judge information favorable his side of the litigation.<sup>6</sup> I assume that the judge cannot directly observe the stakes, but he can infer their values based on the litigation costs incurred by each party. I discuss the signaling aspect of the parties' litigation expenditures in Section 9.

Two economic implications of the utility functions are apparent. First, the utility functions of both parties are characterized by diminishing marginal utility with respect to the value of  $X$ . That property corresponds to the assumption customarily adopted by economists to explain a congeries of observed behavior, including risk aversion. The second implication is that the range of an efficient resolution of the litigation is constrained by the utility functions of the parties. Any value of  $X$  outside the closed interval  $[X_1, X_2]$  is inefficient in the sense that it is not *Pareto* optimal. The difference between the optimal values is the efficient range for adjudication of the numerical value of  $X$  and is symbolized by  $D = X_2 - X_1$ .

#### 5 THE JURISPRUDENTIAL DECISION CRITERION

What decision criterion should the trial judge apply in this case? One possibility is that he should apply a decision rule that actively promotes efficiency. Judge Posner endorsed that proposition when he wrote<sup>7</sup>:

<sup>4</sup> See, e.g. Rasmusen [10, p. 227] The economic inducement to mendacity was recognized by Samuel Johnson when he wrote "Truth is scarcely to be heard, but by those from whom it can serve no interest to conceal it."

<sup>5</sup> The utility functions and the analysis are adapted from Farrel [5]

<sup>6</sup> An illustration of the decisive importance of different utility weights is the conduct of the litigants in the biblical parable after Solomon ordered the infant to be divided.

<sup>7</sup> Posner [9, p. 32.]

*“Efficiency — not necessarily by that name — is an important social value and hence one internalized by most judges, and it may be the only social value that judges can promote effectively, given their limited remedial powers and the value of pluralism in our society. So it should be influential in judicial decision-making”*

An application of the criterion of efficient adjudication in a case of two-party litigation requires a statement of an objective function; viz a statement that can be used to measure the distributive efficiency of the exercise of judicial power. I refer to this as a social welfare function. I assume the judge attempts to achieve distributive efficiency by rendering a decision that maximizes the social welfare function of the litigants appearing before him. It is generally supposed that each individual’s well-being affects social welfare in a symmetric manner, which is to say that the idea of social welfare incorporates a basic notion of equal concern for all individuals. Kaplow and Shavell express the concept as follows:<sup>8</sup>

*“A social welfare function can be any increasing function of individuals’ utilities. In the utilitarian case, for example, the [social welfare function] is the sum  $U_1 + U_2 + \dots + U_n$*

where n is the number of individuals whose welfare is affected by the litigation. In two-party litigation the social welfare function characterizing is the sum of the parties’ individual utility functions.<sup>9</sup>

$$W(X) = U_1(X) + U_2(X) \quad (2)$$

Applying the criterion of distributive efficiency, the judge is assumed to weigh the evidence adduced at trial and render a judgment that maximizes the function  $W(X)$ . The judge will then declare the right to be vested exclusively in a party, as Solomon did, or he may exercise his discretion to assign a numerical value to  $X$ .

## 6 JUDICIAL ESTIMATION OF THE DOMAIN OF LITIGATION

The theory of judicial distributional efficiency requires a balancing test which weighs the utilities of the parties equally, but the court’s assessment of the relative stakes of the parties may tip the balance in one party’s favor. Clearly, the judge’s decision will have distributive consequences measured by the effects on the utilities of the parties of the apportionment of the right in dispute. In this sense it differs from the analysis of legal rules that are concerned exclusively with the efficient allocation of resources.<sup>10</sup> One of the obvious practical obstacles to the implementation of distributive efficiency is that the trial judge does not know the parties’ optimal values of the right in litigation. However, the judge can form a probability distribution governing the optimal value for each party. Thus, the judge can estimate the expected value of each optimum; symbolized by  $E(X_1)$  and  $E(X_2)$ . The difference between these expectations measures the locus of the litigation. I use the symbol  $\bar{D}$  to represent the judge’s expectation of the difference between the parties’ optima;

$$\bar{D} = E(X_2) - E(X_1)$$

The estimated value of  $\bar{D}$  can be used by the judge to identify the boundaries of an efficient declaratory judgment. Henceforth I will refer to  $\bar{D}$  as the domain of the litigation.

## 7 SIGNALING IN LITIGATION

Another obstacle to efficient adjudication is the judge’s ignorance of the relative stakes,  $\theta_1$  and  $\theta_2$ . However, I assume that the judge can estimate the stakes. The judge’s estimation is based on signals sent by the parties during the course of the litigation. The basic idea of signaling is often attributed to Spence [13], who modeled educational attainment as a signal of natural ability. Whatever the merits of Spence’s account of education, his general point regarding credible information transmission is cogent; an individual’s action reliably “signals” that he is of a particular “type” ----- i.e. has particular characteristics or knows particular information ----- when it would not have been in his interest to take that action were he some other type. As a practical matter, the expenditures of money and effort at trial perform an efficient signaling function. The parties to the litigation almost always know more than the judge about the crucial facts, and transmitting the information to the judge is costly to the parties.

<sup>8</sup> Kaplow [7, p. 24]

<sup>9</sup> The definition of the social welfare function in equation (2) implies that the litigation does not generate any external effects on persons not connected with the litigation.

<sup>10</sup> Calabresi and Melamed [4, p. 1098], for example, take the view that “difficult as wealth distribution preferences are to analyze, it should be obvious that they play a crucial role in the settling of entitlements.”

Thus, the effort that a party puts into the trial provides a signal to the judge. A stronger signal increases the probability that the judge will favor the facts as represented by the sender of the signal. Moreover, a stronger signal usually entails greater costs borne by the sender and the judge can usually observe the manifestation of these costs.<sup>11</sup> Sanchirico [11] analyzed the production and interpretation of legal evidence in a world in which parties can and will attempt to mislead the fact finder whenever it is in their interest to do so, and in whatever manner furthers those interests. That kind of behavior necessitates viewing evidence production, to the extent that it is at all effective, as a form of differential cost signaling. Thus, the judge can infer the importance that the sender attaches to the right in litigation. There is a substantial empirical literature confirming that proposition.<sup>12</sup>

The article by Katz [8] developed a formal model to explore how the stakes of the parties effect signaling. One of his results is relevant here:

*“Corollary: A marginal increase in a single party’s stakes in the controversy will lead her to spend more on research and arguments and will increase her probability of winning. Her opponent will spend more if and only if her opponent is the favorite.”<sup>13</sup>*

The judge can observe the quantity and, one presumes, the quality of the research and the advocacy paid for by each party. Moreover, it is assumed that the judge can observe, or at least estimate, the relative income and/or wealth of the parties before him. To the extent that those observations are construed by the judge to signal the relative stakes of the parties, he will use the information to estimate the numerical magnitudes of the relative stakes.<sup>14</sup> I assume that numerical values of the weights inferred by the judge are fixed parameters. That information allows the judge to apply a criterion of efficient adjudication based on the expectation of the social welfare function of a declaratory judgment. Appendix 1 shows that the distributively efficient value of  $X$ , symbolized by  $X^*$ , is given by the equation:

$$X^* = w_1 X_1 + w_2 X_2 \quad (3)$$

The symbols  $w_1$  and  $w_2$  represent the **relative** importance of the right in litigation to each of the parties: i.e.

$$w_i = \theta_i / (\theta_1 + \theta_2) \text{ for } i=1,2.$$

This allows the normalized social welfare function to be written as:

$$W(X) = -w_1 (X - X_1)^2 - w_2 (X - X_2)^2 \quad (4)$$

The value of  $X^*$  is the numerical value that would be assigned to the right by an omniscient judge whose objective is maximization of distributive welfare represented by the function in (4).

### **8 THE DISTRIBUTIVE EFFICIENCY OF A DECLARATORY JUDGMENT**

In the general case, if the judge declares the right to be vested exclusively in Party 1, that party will assign his optimal value  $X_1$ . The value of the resulting social welfare function is symbolized by  $W(X_1)$ . The social welfare function cannot be directly computed by the judge because the optimal values for each party are not known to him. However, the judge can presume that Party 1 will choose the value for  $X$  that maximizes his individual utility. In the specific utility function for Party 1 defined by equation (1), that maximum value is zero. On the other hand, Party 2 will experience a sub-optimal utility because  $X_1 < X_2$ , implying  $U_2(X_1) < U_2(X_2)$ . The judge can form an expectation as to the value of the social welfare function if Party 1 is declared the exclusive holder of the right. That expectation is expressed as:

$$E[W(X_1)] = -w_2 E[(-D)^2] \quad (5)$$

<sup>11</sup> The article by Katz [8] developed an analyzed a model of litigation in which the parties choose the amounts they spend on legal research and argument, given the fact that a legal dispute has arisen. In his model, parties spend resources on legal research to produce arguments that will help influence the court’s decision in their case. The arguments are influential because they alter the likelihood that the decision will be based on a host of minor factors that in the aggregate appear to be random.

<sup>12</sup> Kakalik, et. al. [6, p 648] (“higher stakes are associated with significantly higher total lawyer work hours, significantly higher lawyer work hours on discovery, and significantly longer time to deposition.”) Willging, et. al. [14, p. 533] “the size of the monetary stakes in the case had the strongest relationship to total litigation costs of any of the case characteristics studied.”

<sup>13</sup> Katz [8, p. 136] Katz defines the “favorite” as the party with greater than a 50 percent chance of winning. [p. 131]

<sup>14</sup> I assume that the judge’s determination of the relative weights imputed to each party reflect not only the costs of litigation borne by each, but also the judge’s estimation of the relative wealth and income of the parties. Moreover, I assume that the judge’s estimates of the relative stakes are statistically independent of the parties’ optima.

On the other hand, if the judge decides that the right is exclusively vested in Party 2, the judge can presume that party will assign a value of  $X_2$ . The expected value of the resulting social welfare function is:

$$E[W(X_2)] = -w_1 E[D^2] \quad (6)$$

A comparison of equations (5) and (6) reveals that  $E[W(X_1)] > E[W(X_2)]$  if and only if  $w_1 > w_2$ .

The symbolism translates to mean that the expected social welfare of declaring the right to be vested exclusively in Party 1 will exceed the expected social welfare of declaring the right to be vested exclusively in Party 2 if and only if the relative value of the right to Party 1 is greater. This result can be summarized as a general proposition.

#### PROPOSITION 1

**In an action for a declaratory judgment, if the judge can estimate the relative importance of the right to each party, a declaratory judgment will be distributively efficient if it vests the right in the party where its relative value is the largest.**

PROPOSITION 1 supplies analytical rigor to the definition of efficiency; it explicitly recognizes that the rule assigning the legal entitlement will vest the right in whichever of the two parties values it more. The result is an economic rationalization of Solomon's judgment in the famous child custody case. If the judge can estimate the relative wealth of the parties he can infer the relative stakes of the parties. If, for example, the judge recognizes that the financial net worth of Party 1 is significantly smaller than that of his adversary, and the parties incur similar litigation expenses, the judge will infer that Party 1 is sending a stronger signal than Party 2. The information content of the signal is construed by the judge to mean  $w_1 > w_2$ .

#### 9 DISCRETIONARY JUDGEMENT APPORTIONING THE VALUE OF THE RIGHT IN LITIGATION

I assume that the judge has jurisdictional authority to exercise his discretion to assign the value of the right in issue, thereby precluding exclusive vesting in a party.<sup>15</sup> The judge does not know the optimal value for either party, but he can estimate their expected values. I assume the judge can exercise his discretion to adjudicate a value for  $X$  that is equal to the expected value of  $X^*$ . Let the symbol  $X_j$  represent the expected value of the efficient judgment determining the numerical value of the right in litigation. It is defined by the following equation:

$$X_j = E[X^*] = w_1 E(X_1) + w_2 E(X_2)$$

The judgment  $X_j$  generates an expected value of the welfare function. Appendix 2 shows that the expectation can be expressed as:

$$E[W(X_j)] = -w_1 \text{Var}(X_1) - w_2 \text{Var}(X_2) - w_1 w_2 \bar{D}^2 \quad (7)$$

There are two implications of equation (7) bearing on the efficient judicial exercise of discretion. They are summarized in PROPOSITION 2.

#### PROPOSITION 2

**If the uncertainty of either party's optimum value increases, *ceteris paribus*, the loss of expected welfare is greater for a discretionary judgment than for a declaratory judgment.**

PROPOSITION 2 reflects the loss of jurisprudential efficiency resulting from concealment of private information as to the parties' interests. To the extent that the parties succeed in obscuring their preferred outcomes, they will tend to diminish the efficiency of discretionary judgments.

The signals transmitted by the parties may nullify their efforts to conceal their undisclosed optima. The expenses incurred by each party transmit two kinds of information to the judge. One kind of information, discussed previously, is the information signaling the relative importance the parties assign to the contested right. The second kind of information goes to the value of the right itself. To the extent that the quantity and the credibility of the evidence adduced by the parties (independent of its cost) allows the judge to draw inferences about their private valuations, the variances in expression (7) will be reduced.

<sup>15</sup> The parties may agree by stipulation to confer this power on the trial judge

To take an extreme case, suppose the evidence adduced by both parties allows the judge to infer the actual optima for each, i.e. the judge infers the values  $X_1$  and  $X_2$ . In that case,  $Var(X_1) = Var(X_2) = 0$ . In that case, the apportionment of  $X_j$  which maximizes social welfare is equal to the efficient value  $X^*$  given by equation (3).

### **10 COMPARISON OF AN EFFICIENT DISCRETIONARY JUDGEMENT WITH AN EFFICIENT DECLARATORY JUDGEMENT**

Equation (7) allows a comparison of the distributive efficiency of a discretionary judgment with the distributive efficiency of a declaratory judgment. In order to carry out the comparison, I arbitrarily assume that the declaratory judgment awards the right to Party 1. I assumed he will assign the value  $X = X_1$ . In Appendix 3 I show that the expected welfare function can be written as:

$$E[W(X_1)] = -w_2 \left[ Var(X_1) + Var(X_2) - 2Cov(X_1, X_2) + \bar{D}^2 \right] \quad (8)$$

Equation (8) states that the expected value of a declaratory judgment reflects not only the uncertainty of the trial judge regarding the optima of the parties, but also the extent to which the those optima are correlated, if at all. Let the symbol  $\rho$  represent the correlation coefficient of the parties' optima. Appendix 4 derives an implication of equation (8) expressed by PROPOSITION 3.

#### **PROPOSITION 3**

If the parties' utility functions are quadratic and they assign equal relative importance to the value of the right, a judgment determining the right to be an equally weighted sum of the expected values of the parties' optima is more distributively efficient than a declaratory judgment determining the right to be exclusively vested in either party if and only if:

$$\rho < \frac{1}{4} \frac{[E(X_2) - E(X_1)]^2}{\sqrt{Var(X_2) Var(X_1)}}$$

The result expressed by PROPOSITION 3 establishes a condition that allows a comparison of the efficiency of a declaratory judgment and the efficiency of a discretionary judgment. One of its implications is that if the trial judge is unable (or unwilling) to assess the domain of the litigation, he may be able to infer the relative efficiency of the two judgments if he has adequate information about the correlation between the parties' optimal values of the right in litigation. The judicial criterion is expressed as a corollary to PROPOSITION 3:

#### **COROLLARY TO PROPOSITION 3**

If the optimal values of the parties are negatively correlated and if they assign equal relative weights to the value of the right, then a discretionary judgment apportioning the right as an equally weighted sum of the expected values of the parties' optima is more efficient than a declaratory judgment determining the right to be vested exclusively in either party.

In litigation over some chattel the valuations of the parties are much more likely to be positively correlated. If the positive covariance is very large, the inequality expressed in PROPOSITION 3 will not be satisfied and a declaratory judgment will be the more efficient resolution of the litigation.

An interesting insight to the question of correlated values is found in the recent article by Ayres and Goldblatt [1, p. 128.] They comment as follows:

*"A major source of correlated valuation is the potential exchange value of the entitlement. What tends to be correlated in value is that component for which there is a market value. But market values tend to be relatively observable by judges. It is the litigants' idiosyncratic, nonmarket values that are less likely to be observed by judges and less correlated."*

A central proposition of Ayers' recent book [2] is that liability rules can be viewed as call options --the option for any potential infringer to take the entitlement if he pays the exercise price of the legally fixed damages. The relevance of Ayers' option theory to the results in this paper is that he shows how liability rules allow imperfectly informed judges to delegate the allocative decision to disputants with private information. One of Ayers' findings dovetails nicely with the result summarized in PROPOSITION 3 above. Here is Ayers' statement [2, p. 7]

“A basic principle of option pricing is that underlying volatility makes options more valuable. As applied to liability rules, this suggests that the litigant who, from the judges’ perspective, has the more speculative valuation is likely to be the more efficient chooser. Until now, judges have focused too much on the mean [sic.] of the litigants’ valuations. Option theory suggests that the variance is more important to allocative efficiency.”

The relevance of PROPOSITION 2 and PROPOSITION 3 to Ayers’ principle is reasonably transparent. If the judge estimates the variances of the parties’ optima (i.e. the *volatility* mentioned by Ayres) the propositions developed in this paper suggest that a declaratory judgment vesting the right in the party with the larger variance will lead to a more efficient apportionment of the contested right. In the most commonly occurring litigations where the correlation of the parties’ optima is strongly positive, PROPOSITION 3 implies that if the judge’s estimates of the variances are large, the inequality in the proposition will be reversed. In that case, declaratory judgment vesting the right exclusively in one party will be the more distributively efficient adjudication.

## 11 CONCLUDING REMARKS

This paper considers cases where litigants contesting a legal right are unable to negotiate with each other to apportion the right, either *ex ante* the litigation or *ex post*. The paper considers the question of what criterion the court might use to decide the issue. The judge does not know the parties’ optima and regards them as random variables. If the judge estimates the variances of the parties’ optima, the propositions developed in this paper suggest that a declaratory judgment vesting the right in the party with the larger variance will lead to a more distributively efficient apportionment of the contested right.

A main conclusion of the paper is that if the judge’s uncertainty of either party’s optimum value increases, *ceteris paribus*, the loss of expected welfare is greater for a discretionary judgment than for a declaratory judgment. In many cases, the optimal values of the parties are negatively correlated. In those kinds of cases, if the economic signals transmitted by the litigants imply (to the judge) that they assign equal relative weights to the value of the right in issue, a discretionary judgment apportioning the right as an equally weighted sum of the expected values of the parties’ optima is more distributionally efficient than a declaratory judgment determining the right to be vested exclusively in either party.

## APPENDIX 1

Maximization of the welfare function entails finding the derivative of expression (2) with respect to  $X$ , setting it equal to zero, and solving for the solution value. The derivative is:

$$\frac{dW(X)}{dX} = -2\theta_1(X - X_1) - 2\theta_2(X - X_2)$$

Setting  $dW/dX = 0$  and solving for the value of  $X$ , symbolized by  $X^*$ , we have:

$$X^* = \frac{\theta_1}{\theta_1 + \theta_2} X_1 + \frac{\theta_2}{\theta_1 + \theta_2} X_2$$

Let  $w_1 = \frac{\theta_1}{\theta_1 + \theta_2}$  and  $w_2 = \frac{\theta_2}{\theta_1 + \theta_2}$ . It is obvious that  $w_1 + w_2 = 1$ , thus establishing equation (3) in the text.

## APPENDIX 2

Welfare as a function of the judicial apportionment of the right is written as

$$W(X_J) = U_1(X_J) + U_2(X_J) = -w_1(X_J - X_1)^2 - w_2(X_J - X_2)^2 \quad A2.1$$

The expected value of the social welfare function pursuant to the judge’s determination of  $X_J$  is written as:

$$E[W(X_J)] = -w_1 E[(X_J - X_1)^2] - w_2 E[(X_J - X_2)^2] \quad A2.2$$

Each quadratic term can be expanded as follows for  $i = 1, 2$

$$E[(X_J - X_i)^2] = X_J^2 + E[X_i^2 - (E(X_i))^2] - 2X_J E(X_i) + [E(X_i)]^2 \quad A2.3$$

The middle term on the right-hand side of A2.3 is recognized as the variance of  $X_i$  around the judge's expectation of its optimal value. Substituting the variances into equation A2.3 and collecting terms into the closed form quadratic expression, we have:

$$E[(X_j - X_i)^2] = Var(X_i) + [X_j - E(X_i)]^2 \text{ for } i = 1, 2 \quad A2.4$$

Substituting the equations in A2.4 into equation A2.2, the expected value of the social welfare function can now be written as:

$$E[W(X_j)] = -w_1 Var(X_1) - w_2 Var(X_2) - w_1 [X_j - E(X_1)]^2 - w_2 [X_j - E(X_2)]^2 \quad A2.5$$

The judicial determination of  $X_j$  can be substituted into the expressions in the brackets on the right-hand side of equation A2.5. Exploiting the fact  $w_1 + w_2 = 1$ , we have the results:

$$\begin{aligned} X_j - E(X_1) &= w_2 [E(X_2) - E(X_1)] = w_2 \bar{D} \\ X_j - E(X_2) &= w_1 [E(X_1) - E(X_2)] = w_1 (-\bar{D}) \end{aligned} \quad A2.6$$

The right hand sides of equations in A2.6 are fixed parameters. Squaring them, substituting them into equation A2.5 and collecting terms we have:

$$E[W(X_j)] = -w_1 Var(X_1) - w_2 Var(X_2) - (w_1 w_2^2 + w_2 w_1^2) \bar{D}^2 \quad A2.7$$

The weights appearing in the parenthetical term on the right side of equation A2.7 can be factored as  $w_1 w_2^2 + w_2 w_1^2 = w_1 w_2 (w_1 + w_2) = w_1 w_2$ . This produces the result in the text.

### APPENDIX 3

Expanding the domain of litigation in equation (4) the text and applying the expectation operator we have the result:

$$E[W(X_1)] = -w_2 \left[ E(X_1^2) + E(X_2^2) - 2E(X_1 X_2) \right] \quad A3.1$$

The expectation of the parties' optima can be expressed in terms of the covariances of the values:

$$E[X_1 X_2] = Cov(X_1, X_2) + E(X_1)E(X_2) \quad A3.2$$

The expression in A3.2 can be substituted into A3.1. Also, the variances of the optimal values can be substituted in A3.2. The result is:

$$E[W(X_1)] = -w_2 \left[ Var(X_1) + Var(X_2) - 2Cov(X_1, X_2) + E(X_1)^2 + E(X_2)^2 - 2E(X_1)E(X_2) \right] \quad A3.3$$

The last three terms in equation A3.3 can be compressed to a binomial expression to yield equation (8) in the text.

### APPENDIX 4

The proof of PROPOSITION 3 proceeds by deriving the necessary and sufficient conditions to establish that the expected values of the efficient discretionary welfare function exceeds the expected value of the efficient declaratory judgment, under the condition stated by the proposition. The inequality between the expected functions is expressed as:

$$E[W(X_j)] > E[W(X_i)] \text{ for } i = 1, 2 \quad A4.1$$

The left side of the inequality can be represented by the expression (8). The right side of the inequality can be represented by expression (9). Thus the inequality above will be satisfied if and only if the following inequalities are satisfied for  $i=1, 2$ :

$$\begin{aligned} -w_1 Var(X_1) - w_2 Var(X_2) - w_1 w_2 \bar{D}^2 &> \\ &- w_i \left[ Var(X_1) + Var(X_2) - 2Cov(X_1, X_2) + \bar{D}^2 \right] \end{aligned} \quad A4.2$$

Let  $i=2$ . (The algebraic result will be the same when  $i=1$ .) Then the inequality in A4.2 can be simplified to:

$$-w_1 w_2 \bar{D}^2 - w_1 Var(X_1) > -w_2 \bar{D}^2 - w_2 Var(X_1) + 2w_2 Cov(X_1, X_2) \quad A4.3$$



Collecting terms as common factors in the inequality in A4.3 we have:

$$(1 - w_1)w_2 \overline{D}^2 > (w_1 - w_2)Var(X_1) + 2w_2 Cov(X_1, X_2) \quad \text{A4.4}$$

If the parties assign equal importance (i.e. equal utility weights) to the value of the right, then  $w_1 = w_2 = \frac{1}{2}$ .

In that case, the inequality in A4.4 reduces to

$$\frac{\overline{D}^2}{4} > Cov(X_1, X_2). \text{ The inequality is expressed as PROPOSITION 3.}$$

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