Optimal Bailout Policy–An Analysis Focusing on the Regulation of Collective Banking Behavior

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Abstract

Bank regulation theory has evolved to focus on the supervision of the collective behavior of banks. Parallel investing theory, which presents parallel investing as a collective strategy of all banks, is understood as an extension of herding theory and is used to discuss optimal bailout policies. First we expand the traditional one-dimensional regulatory objective, social welfare maximization, to 3d to include efficiency and risk factors. Based on this hypothesis, we conclude that an optimal bailout policy capable of achieving the 3d objectives does not exist. We term this phenomenon objective realization–inconsistency: regulators can only balance the 3d objective and make relatively effective policies. Finally, we compare financial derivatives with bailout policy in the banking regulatory role. Under certain conditions, financial derivatives can play a very active role in banking regulation.

Keywords: Collective banking behavior; Parallel investing; Objective-inconsistency; bailout; financial derivatives

1. Introduction

Bank regulators can use their privileges to devise policies to restrain banks' behaviors. Influence is, ostensibly, one-way: from the regulator to the banks. As long as the target is clear, the regulator should be able to formulate pertinent policies capable of solving banks' problems. As a matter of fact, however, the regulator and the banks are matched in the game: the object the regulator faces is not individual Banks, but the banking system and the behavior of banks can also influence the regulator. The linkages between banks make bank regulation more complex wherein the regulator needs to consider both the effect of regulations on both single banks and to the banking system. We argue that the regulation of collective behavior is more important than regulation of individual banks. 'Collective behavior' here refers to the interaction between Banks, including parallel behavior and competition. The parallel behavior we focus on is not realized by a contract, but rather by an investment strategy pursued across banks. Motivated by profit maximization, banks 'naturally' orientate toward a parallel strategy. The effect of a single bank's reaction is small because there are large numbers of banks in the market. However, the collective behavior of banks exerts a huge influence on regulators. This is especially the case when banks choose parallel investing to make the correlation of their assets high.

In such situations problems may become systemic and the regulator may be forced to bailout banks. Bailout policy is actually a subsidy to banks. When banks anticipate and expect bailouts they will spontaneously adopt a parallel strategy. The bailout policy therefore becomes banks' protective umbrella, which can increase systemic risk, and even lead to financial crisis affecting the whole of society. Extant theories refer to this umbrella as the "too many to fail guarantee" and it is also one source of moral hazard. The competition means that banks lower the correlation of their assets, optimizing investment to achieve further development. In this competition, banks try to beat each other in order to reap more benefits. As a whole, cooperation and competition both generate profits for banks. It is policy that decides which strategy is chosen by banks. The optimal bailout policy is the core problem discussed in our paper. The problem is modeled as a game. The players are one regulator and two identical banks. There are two periods and information is complete and symmetric. The components of the regulator's 3d objective vector are defined as social welfare, the output of banking sector and the risks. Banks choose whether to adopt a parallel strategy and evaluate the risk. Banking asset transaction includes loans and investments in varieties of bonds. In this paper we define banking investment as the whole asset transaction. That means a bank's asset is a portfolio of various loans and investments in bonds.

The return of any portfolio is expressed as a variable subject to binomial distribution. **Parallel investing** refers to the practice of banks loaning to the same industry or purchasing the same bonds so that their assets are identical or at least closely related. In this case they not only take the same risk, but also get the same or similar returns (which may be nil). If banks choose to invest independently there is no correlation between their assets. We use backwards introduction to analyze this problem. First, we discuss portfolio selection by banks in the second period, then the regulator's strategy in this period. We conclude that although bailout is the sub-optimal strategy, it is inevitable when both banks fail. Next, we discuss how banks select investment in the first period according to the regulator's strategy. Finally, we try to discover the optimal bailout policy. The analysis evidences a negative correlation between the strictness of regulation and the probability of problems. However, strict regulations also reduce banking output and are harmful to future banking development. In contrast, relaxing the regulation could provide more opportunities for banking to develop, but would lead to higher risk.

In addition, assessing how such policy affects social welfare is a particularly complicated process. The policy should balance these three aspects. We term this issue objective realization–inconsistency. This means regulators cannot maximize all three components simultaneously. There exists only the potential for a balanced policy that relatively achieves the objective. Financial derivatives are also discussed, in particular, credit-default swaps, to establish if financial derivatives could help lower risk and stabilize the banking sector. Compared with bailout policy financial derivatives appear to be able to play a very active role in banking regulation. The remainder of the paper is structured as follows. Section 2 provides a review of the literature. Sections 3 and 4 present the model and the analysis. Section 5 discusses the effect of financial derivatives on banking regulation. Section 6 provides the conclusion and policy suggestions. Proofs that are not in the main text are contained in the Appendix.

2. Literature Review

Collective strategy can be realized not only by parallel investing but also via interbank transactions. An individual bank's behavior can transform into collective behavior which, in turn, can influence the regulator's strategy through interbank transaction. Rochet and Tirole (1996) discuss how to prevent systemic risk caused by interbank transaction. Traditionally, bailout has been seen as an effective method of preventing systemic risk. However, it is also associated with moral hazard. Centralizing banks' liquidity management lowers systemic risk by reducing interbank linkage, but also reduces the flexibility of the interbank market. Rochet and Tirole argue that a decentralized operation of interbank lending must incorporate peer monitoring to preserve flexibility and simultaneously control systemic risk. Freixas, Parigi and Rochet (2000) argue that interbank connections enhance the "resiliency" of the system to withstand the insolvency of a particular bank, because a proportion of the losses on one bank's portfolio is transferred to other banks through interbank agreements. But such a network of cross-liabilities may allow an insolvent bank to continue operating through the implicit subsidy generated by interbank credit lines, thus weakening the incentives to close inefficient banks. Individual behavior is thus transformed into collective behavior through the network and thus influences the regulator's strategy. These issues are similar to those addressed here, although we propose an alternative way of realizing collective behavior.

Bailout is not ex-ante optimal, because it gives rise to moral hazard. But when a crisis occurs it is the inevitable choice, i.e. the ex-post optimal strategy. Since bailout is inevitable, it is important to design the optimal bailout policy. Freixas (1999), Ringbom, Shy and Stenbacka (2003), and Aghion, Bolton and Fries (1999) calculate different optimal bailout policies stressing different objectives using cost-benefit analysis methods. In particular, Ringbom, Shy and Stenbacka discuss a two banks model involved in interbank transaction, and analyze how the banks' interactions influence regulation. Our model is influenced by Acharya (2007) whose theory helps analyze the time-inconsistency problem. Acharya (2009)[1] develops a new theory to explain systemic risk implicating herding. Herding refers to banks loaning to the same corporate sector. In this case, the correlation of different banks' returns from loans is high. When firms default on their loans systemic risk will rise. Acharya (2009) analyzes what policies force banks to loan to different sectors in order to lower systemic risk. Earlier research by Acharya and Yorulmazer (2007) and Acharya and Yorulmazer (2008) complement Acharya (2009). They successively discuss the time-inconsistency problem and liquidity provision which compensates for the defects of traditional bailout policy.

Parallel investing is an extension of herding theory. We maintain that other aspects of asset operation, in addition to loaning to same industries, can make the correlation between different banks' returns high. Parallel investing is a whole assets operation.

[1] The first draft of Acharya (2009) was finished in 2000

We consider parallel investing as a collective strategy and in the following discuss the objective-inconsistency problem. Financial derivatives are perceived to be the most important financial innovation in terms of transferring banking risks to other investors and reducing the probability of default. Consequently, banks achieve more long term profits. Silber (1983) argues that most financial innovations try to surmount or circumvent various endogenous and exogenous constraints affecting the behaviors of financial institutions. Duffee and Zhou (2001) investigate the effect of financial derivatives on the banking sector. They argue that financial derivatives can disperse banking credit risks and lower the probability of crisis caused by non-performing assets. Based on their research, we introduce financial derivatives in our model to analyze the effect on banking regulations.

3. The Model

Our model stems from Acharya and Yorulmazer (2007). Their paper presents the 'spirit' of the collective behavior of banks. To accommodate an extension of herding theory, we make a key change in the model. We consider an economy with three dates—t = 0, 1, 2, two periods—t=0 to 1 and t=1 to 2. The players in the game are two identical banks, and one regulator (typically the government or the central bank). In additional, there are adequate amount of depositors and outside investors. Each bank can borrow from a continuum of depositors of measure 1. Bank owners as well as depositors are risk-neutral. Deposits take the form of a simple debt contract with maturity of one period. Suppose r is the promised deposit rate that is not contingent on investment decisions of the bank or on realized returns. The information is complete and symmetric.

Banking asset transactions include loans and investments in a variety of bonds. In this paper we define banking investment as the whole asset transaction. That means a bank's asset is a portfolio of various loans and investments in bonds. The return of any portfolio is expressed as a variable subject to binomial distribution. Parallel investing means that different banks loan to the same industry or purchase the same bonds so that their assets are identical or closely related. They therefore not only take the same risk, but are also promised the same returns (which may, of course, be nil). If banks choose to invest independently, there is no correlation between their assets.

Suppose R is the promised return on a portfolio. The bank gets R with probability α_t. $R = R(\alpha_t)$, $R^{'} < 0$, $R^{''} < 0$.[2] Banks choose different portfolios, i.e. R to maximize expected profits. $1 - \alpha_t$ is the probability that banks get nothing: it therefore represents the risk of the investment. μ is the discount factor.

With deposit insurance, the expected profit in one period is

$$\alpha_t(R(\alpha_t) - r)$$

When return is 0, the bank is in default and the regulator has to pay the deposits. So in this case, the profit is 0.

In addition to banks and depositors, there are outside investors who have funds to purchase banking assets were these assets to be liquidated. However, outsiders do not have the skills to generate the full value from banking assets.[3] Here we suppose that the outsiders only generate σR while bank owners generate R.[4] Outsiders are less inefficient than the bank owners.

Finally, we suppose the regulatory objective is a 3d vector. The components of the regulator's 3d objective vector are defined as social welfare, namely the output of banking sector net of regulatory costs, the output of banking sector and the risks. The risks here include one bank's risk and systemic risk. The regulatory costs refer to the social costs caused by the regulator's operation. We denote the 3d objective vector as (W, Π, Ri). In general, the literature assumes that the regulator's objective is merely the maximization of social welfare. This assumption indicates that the regulator is risk-neutral too. The regulator's risk aversion tendency is not invariable. When financial markets are stable, the regulator chooses deregulation to some extent. Conversely, when the conditions are tough the regulator chooses to intensify regulation to maintain stability. Moreover, the regulator behaves differently from consumers, so we cannot simply assume that the regulator is either attracted to risk or risk averse. Consequently, we define the objective as a 3d vector.

When the first period ends, if the bank gets the promised return R, it is allowed to operate for one more period and make the second period investment.(it indicates that $R > r$). If the bank does not achieve the promised return it is in default.

[2] This is the key change to the model of Acharya and Yorulmazer(2007). They assume that the return and risk of an individual bank are invariable.

[3] Diamond and Rajan (2001) assume a distinction in skills between bank managers. Acharya and Yorulmazer(2007) assume a distinction in skills between bank managers and outside investors.

[4] Acharya and Yorulmazer(2007) denote the inefficiency loss as a reduction to return. We use a coefficient to reflect this loss as the risk is variable.

In this case, the regulator decides whether to allow the failed bank to be acquired by a surviving bank (assuming one exists), to liquidate the failed bank's assets to outside investors, or to keep the bank open via bailout. We assume that only after acquiring the failed bank can the surviving bank absorb the failed bank's deposits. The deposits are fully insured in the first and second periods. The provision of funds to pay off failed deposits, net of any proceeds from the sale of a failed bank's assets, entails social costs.[5] These social costs not only include the funds needed to pay off those deposits termed direct costs, but also involve the related losses of GDP both during and after the crisis. Caprio and Klingebiel (1996) provide us with the data of 29 banking crisis cases in 26 countries since 1970. According to their research the direct costs were enormous in many countries. Between 1981 and 1983 a series of Chilean bailout policies cost about 41.2% of GDP. Between 1984 and 1991 the costs of bailouts, as a percent of GDP, were 16.8% for the US and 5-10% for certain European countries. Most seriously, long term real output losses always accompany banking crises. Boyd, Kwak and Smith (2005) conducted a quantitative analysis of the real output losses related to banking crises, factoring-in losses accrued not only during but also after each crisis. They report a correlation between direct costs and real output losses of 0.757 at the 1% confidence level.

Although this research shows that there is high correlation between direct costs and real output losses, it does not provide any explanation about a possible causal relationship. In our analysis we assume that the social costs of funds provision only include the direct costs because there is no certain evidence for attributing output losses to funds provision. The financing function of banking is broken during the crisis, which is one of the most important reason for output losses. The direct costs and the real output losses are both the results of crisis, which accounts for their relationship naturally. In addition, our model does not involve the output factors, so it is unreasonable to simply add output losses to funds provision. Finally, there are only two periods in our model, while the output losses always linger on long after a crisis is over. Output losses will exaggerate the costs of funds provision in our model. In particular, when a failed bank is bailed out, the regulator has to pay off the entire deposits.

Finally, we assume that the regulator dilutes the equity share of bank owners in a bailed out bank. That means a proportion of the profits is acquired by the regulator after the bank's second period operation. Suppose the propotion is β. When the regulator is the central bank, the bailout could be realized by loans to the failed banks. The penalty is the higher interest rate which is similar to the dilution of the equity share.

4. Analysis

We use backwards induction to analyze the optimal strategy of banks and the regulator.

4.1 The bank's strategy in the second period (t=1 to 2)

First, we analyze the bank's optimal strategies in different cases in the second period. In the first period, a bank can either succeed or fail. If the bank succeeds it proceeds to operate in the next period. Otherwise, its assets will be purchased by the other bank and operated by another bank owner or by outside investors and operated by outsiders, or be bailed out.

A. The bank is operated by the bank owner

Because it is the second period of a two-period game, there will be no subsequent investment: whether to choose parallel investing is therefore irrelevant. The bank owner invests to maximize the expected profits for the current period. The objective function is

$$\max \alpha_2(R(\alpha_2) - r)$$

The first order condition w.r.t. α_2 can be expressed as:

$$R(\alpha_2) - r + \alpha_2 R'(\alpha_2) = 0$$

Here, let $g(\alpha) = R(\alpha) - r + \alpha R'(\alpha)$, $g(\alpha)$ is strictly decreasing in α. There exists a unique value $\alpha_2 = \alpha^*$ which satisfies the first order condition. Let $\pi^* = \alpha^*(R(\alpha^*) - r)$. $\alpha_2(R(\alpha_2) - r)$ is increasing then decreasing in α_2, $\alpha_2 \in (0,1)$. α^* is the maximum point. In the second period, the bank owner chooses the risk of $1 - \alpha^*$. The expected profit is π^*. (See proof 1 in Appendix)

B. The bank is operated by the outsider

The objective function of the outsider is:

$$\max \alpha_2(\sigma R(\alpha_2) - r)$$

The first order condition w.r.t. α_2 is:

$$\sigma R(\alpha_2) - r + \alpha_2 \sigma R'(\alpha_2) = 0$$

There exists a unique value $\alpha_2 = \alpha^\#$ satisfying the first order condition. Let $\pi^\# = \alpha^\#(R(\alpha^\#) - r)$. Compared with π^* and α^*, $\pi^* > \pi^\#$, $\alpha^* > \alpha^\#$. (See proof 2 in Appendix)

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[5] These costs may include the expenditure of the funds we call the direct costs and the related real output losses.

The bank owner assumes lower risk and obtains more expected profits than the outsider.

C. Purchasing the failed bank

Suppose the failed bank's assets are sold at price p, $p \leq \mu\pi^*$ (no bank is willing to purchase the assets if $p > \mu\pi^*$,). Moreover, when both banks fail, $p \leq \mu\pi^\#$ (no outsider is willing to purchase the assets if $p > \mu\pi^\#$). First, we assume that p is invariable and satisfies all the conditions above. Let $k = \frac{\pi^* - p/\mu}{\pi^*}$. k represents the benefits brought by the competition. The surviving banks benefit from the competition by beating weak ones.

Lemma 1 The successful bank is willing to purchase the failed one. On purchasing the failed bank, the outsider takes higher risk $\alpha^* > \alpha^\#$ and less profits $\pi^* > \pi^\#$ than the bank owner.

Proposition 1 The bank's optimal strategy in the second period is

A. If the bank is operated by the bank owner, the investment risk is $1 - \alpha^*$, and the expected profit is $\pi^* = \alpha^* \left(R(\alpha^*) - r\right)$.

B. If the bank is operated by the outsider, the investment risk is $1 - \alpha^\#$, and the expected profit is $\pi^\# = \alpha^\# \left(\sigma R(\alpha^\#) - r\right)$.

C. The surviving bank is willing to purchase the failed bank's assets after the first period and receives extra profits $\Delta\pi = \pi^* - \pi^\#$ in the second period.

4.2 The regulator's strategy at t=2

If the bank succeeds in investment in the first period, the regulator will not intervene.

If the bank fails, the regulator's optimal strategy is to sell the bank's assets to the successful one (if existent). In this second period, the social welfare, the output of the failed bank and its risk are

$$W_2 = 2\mu\pi^* - 2r, \pi = \pi^*, \alpha_2 = \alpha^*$$

If the outsider purchases the bank's assets:

$$W_2 = \mu\pi^* + \mu\pi^\# - 2r, \pi = \pi^\#, \alpha_2 = \alpha^\#$$

If the regulator chooses to bailout the failed bank:

$$W_2 = 2\mu\pi^* - 2r, \pi = \pi^*, \alpha_2 = \alpha^*$$

According to the above analysis, selling the failed bank's assets to the successful bank and bailout are both better than selling to the outsider. Selling to the successful bank is indifferent with bailout. However, if the regulator bails out the failed bank, it can only achieve the proportion of profits when the second period ends. When the banking crisis occurs the regulator prefers to replenish funds to cope with unexpected problems as soon as possible, so selling to the successful bank is better than bailout. Although selling to the outsiders offers the regulator the opportunity to realize funds faster than with a bailout, bailout is superior to selling to the outsiders because the 3d objective is the primary concern. In addition, the efficiency losses experienced by the outsider are not restricted to just one period. Bailout is also better in reality when there exists more than two periods.

Proposition 2 At t=1, the regulator's strategy is

i. If the bank succeeds in investment in the first period, the regulator will not intervene.
ii. If the bank fails, the strategy is selected in the following order:
   1) Selling the failed bank to the successful one
   2) Bailout the failed bank
   3) Selling the failed bank to the outsiders

Because of the inefficiency of the outsiders, the regulator has to bail them out when two banks fail together. Aware of this situation, the banks have an incentive to parallelly invest to force the regulator to bail them out when they fail together. We have assumed that p is invariable and it reflects the benefits brought by the competition. The lower p is the more benefits the successful bank will achieve from the purchase. So a relatively lower p provides banks with an incentive to compete but not to parallelly invest. However, the regulator has to replenish the funds during the banking crisis as soon as possible. As a result p cannot be set too low. A full discussion of the pricing problem is beyond the remit of this study, but is clearly relevant.

4.3 The bank's strategy in the first period

We assume that the banks can predict the regulator's plan in the second period. Next, we want to find out: the banks' strategy in the first period, the optimal β and the equilibrium of the game. The bank's strategy includes not only the selection of the investment portfolio, but also whether to parallelly invest. Accordingly, the banks make a plan to maximize the whole profits in the two periods. If there exists no bailout, for banks $\beta = 1$. Whether banks choose to parallelly invest, the objective function is

$$\max \alpha_1(R(\alpha_1) - r) + \mu(\alpha_1\pi^* + (1 - \alpha_1) \times 0 \times \pi^*)$$

The first order condition is

$$R(\alpha_1) + \alpha_1 R'(\alpha_1) = r - \mu\pi^*$$

We denote α_1 which satisfies the first order condition as α_1^{1*}. Let $\pi^{1*} = \alpha_1^{1*}(R(\alpha_1^{1*}) - r) + \mu\alpha_1^{1*}\pi^*$.

When two banks in the model choose to parallelly invest, the probability of joint failure is $1 - \alpha_1^{1*}$. When two banks choose to invest individually, this probability is $(1 - \alpha_1^{1*})^2$. Obviously, $(1 - \alpha_1^{1*})^2 < (1 - \alpha_1^{1*})$. In this two banks model, we can use the probability of joint failure to measure the systemic risk. The more similarity there exists between two banks' assets, the more likelihood the banks will fail together. To lower the systemic risk, two banks investing independently is the ideal equilibrium for the regulator.

However, we have seen that bailout is better than selling the failed banks to outsiders. That means possibly$\beta < 1$. The bank's strategy will be complex.

A. Two banks choose to invest parallelly

Two banks choose to invest parallelly. They then select a portfolio to maximize each of their profits. With the regulator's strategy in the second period, their common objective function is

$$\max \alpha_1(R(\alpha_1) - r) + \mu(\alpha_1\pi^* + (1 - \alpha_1)(1 - \beta)\pi^*)$$

The first order condition is

$$R(\alpha_1) - r + \alpha_1 R'(\alpha_1) = -\mu\beta\pi^*$$

We denote α_1 which satisfies the first order condition as α_1^{2*}, $\alpha_1^* < \alpha_1^{2*} < \alpha_1^{1*}$. Let $\pi^{2*} = \alpha_1^{2*}(R(\alpha_1^{2*}) - r) + \mu(\alpha_1^{2*}\pi^* + (1 - \alpha_1^{2*})(1 - \beta)\pi^*)$. $\alpha_1^{2*} < \alpha_1^{1*}$ indicates that bailouts encourage banks to raise risk.

From the first order condition equation, it can be observed that α_1^{2*} is decided by β. So α_1^{2*} can be expressed as a function of β, $\alpha_1^{2*} = \alpha_1^{2*}(\beta)$. Actually, α_1^{2*} is increasing in β and π^{2*} is decreasing in β (See proof 3 in Appendix).

Lemma 2 When parallelly investing, as β increases two banks lower their risks. As a consequence, the systemic risk and the expected profits of a single bank both decrease.

The regulator can set high β to control the banks' behavior. On the one hand high β forces banks to lower the risk when they parallelly invest; but on the other hand decreased profits provide banks with an incentive to deviate from the parallel investing strategy.

In this case, the aggregate outputs of the banking sector are

$$\Pi(\beta) = 2\alpha_1^{2*}(\beta)\left(R\left(\alpha_1^{2*}(\beta)\right) - r\right) + 2\mu\pi^*$$

$\Pi(\beta)$ is decreasing in β (see proof 4 in Appendix). The heavier the penalty is, i.e. the higher the β is, the less the aggregate outputs are.

The social welfare is

$$W(\beta) = 2\pi^{2*}(\beta) + 2\left(1 - \alpha_1^{2*}(\beta)\right)(\mu\beta\pi^* - r)$$

We rewrite the equation as:

$$W(\beta) = 2\alpha_1^{2*}(\beta)\left(R\left(\alpha_1^{2*}(\beta)\right) - r\right) + 2\mu\pi^* - 2\left(1 - \alpha_1^{2*}(\beta)\right)r$$

We could see how $W(\beta)$ varies in β by the derivative of $W(\beta)$, $W'(\beta) = 2\alpha_1^{2*\prime}(\beta)R\left(\alpha_1^{2*}(\beta)\right)\Big(1 -$ $ER\alpha\alpha12*\beta=2-\mu\beta\pi*+r\alpha12*'\beta$. $ER\alpha$ is the $W\beta$ elasticity to α. $ER\alpha\alpha12*\beta$ is increasing in β. Theoretically, $\beta = \frac{r}{\mu\pi^*}$ is the maximum point of $E_{R\alpha}\left(\alpha_1^{2*}(\beta)\right)$. Let $\bar{\beta} = \frac{r}{\mu\pi^*}$. If $\beta > \bar{\beta}$, $W'(\beta) < 0$, $W(\beta)$ is decreasing in β; If $\beta < \bar{\beta}$, $W'(\beta) > 0$, $W(\beta)$ is increasing in β (see proof 5 in Appendix).

Lemma 3 When two banks parallelly invest:

(1) The aggregate outputs of banking sector are decreasing in β.

(2) The variation tendency of social welfare $W(\beta)$ in β is:

  i. If $\bar{\beta} < 1$, namely $r < \mu\pi^*$, $\beta < \bar{\beta}$ and $W(\beta)$ is increasing in β.If $\beta > \bar{\beta}$, $W(\beta)$ is decreasing in β. $\beta = \bar{\beta}$ is the maximum point of $W(\beta)$.

  ii. If $\bar{\beta} \geq 1$, namely $r \geq \mu\pi^*$, $W(\beta)$ is increasing in β.

B. The incentive to deviation

If one bank deviates from the parallel investing strategy when the other one takes $\alpha_1 = \alpha_1^{2*}$. To maximize its profits, the deviating bank's objective function is

$$\max \alpha_1(R(\alpha_1) - r) + \mu[\alpha_1(\alpha_1^{2*}\pi^* + (1 - \alpha_1^{2*})(1 + k)\pi^*) + (1 - \alpha_1)(\alpha_1^{2*} \times 0 \times \pi^* + (1 - \alpha_1^{2*})(1 - \beta)\pi^*)]$$

The first order condition is

$$R(\alpha_1) - r + \alpha_1 R'(\alpha_1) = -\mu(\beta\pi^* + \alpha_1^{2*}(1-\beta)\pi^* + (1-\alpha_1^{2*})k\pi^*)$$

The unique solution of this equation is $\alpha_1 = \alpha_1^{3*}$, $\alpha_1^{3*} > \alpha_1^{2*}$. Let $\pi^{3*} = \alpha_1^{3*}(R(\alpha_1^{3*}) - r) + \mu[\alpha_1^{3*}(\alpha_1^{2*}\pi^* +$ *(1−α12∗)(1+k)π∗)+(1−α13∗)1−α12∗1−βπ∗. α13∗′β>0, π2∗′β<π3∗′β<0,* and *∂2π2∗∂β2>0,* $\frac{\partial^2\pi^{3*}}{\partial\beta^2} > 0$. Meanwhile, $\pi^{2*}(0) > \pi^{3*}(0)$, $\pi^{2*}(1) < \pi^{3*}(1)$. There exists a unique β^*, if $0 < \beta^* < 1$, $\beta \le \beta^*$, the profits from parallel investing are higher than those from deviation. If $\beta > \beta^*$, the profits from deviation are higher (see proof 6 in Appendix).

Lemma 4 As β increases, the deviating bank lowers its risk and its expected profits are decreased. Compared with the case of parallel investing, the decrease in the profits of the deviating bank in β is slower. The regulator can force the banks to deviate by setting β higher than β^*.

The variations tendency of $\pi^{2*}(\beta)$ and $\pi^{3*}(\beta)$ in β are demonstrated by Figure 1.

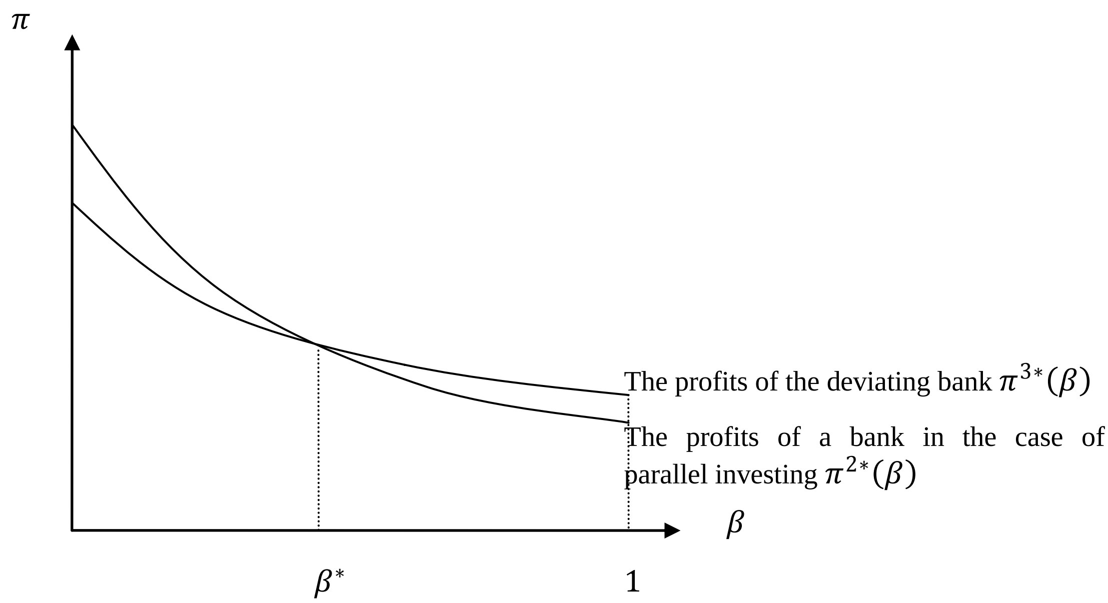

Figure 1

If $\beta \le \beta^*$, no bank has an incentive to deviate from parallel investing and the return of the assets of the two banks are in high correlation, $R = R\left(\alpha_1^{2*}(\beta)\right)$. Theoretically, if $\beta > \beta^*$, a bank has an incentive to deviate. According to the figure above the risk and the expected profits decrease together, $\alpha_1^{2*'}(\beta) > 0$, $\alpha_1^{3*'}(\beta) > 0$, $\pi^{2*'}(\beta) < \pi^{3*'}(\beta) < 0$. The regulator has to sacrifice the outputs of the financial sector for lower risks.

C. Independent investment choices

The regulator can set $\beta > \beta^*$to avoid the parallel investing of two banks. Then two banks have to adopt independent asset portfolios. That means the collective behavior of the banks would be replaced with competition, which is preferred by the regulator.

Now we will analyze the possible equilibrium of the game. The objective function of one bank is

$$\max \alpha_1(R(\alpha_1) - r) + \mu[\alpha_1(\widetilde{\alpha_1}\pi^* + (1-\widetilde{\alpha_1})(1+k)\pi^*) + (1-\alpha_1)(\widetilde{\alpha_1} \times 0 \times \pi^* + (1-\widetilde{\alpha_1})(1-\beta)\pi^*)]$$

$\widetilde{\alpha_1}$ is the risk of assets chosen by the other bank. The first-order condition is

$$R(\alpha_1) - r + \alpha_1 R'(\alpha_1) = -\mu(\beta\pi^* + \widetilde{\alpha_1}(1-\beta)\pi^* + (1-\widetilde{\alpha_1})k\pi^*)$$

Similarly, the first-order condition of the other bank is

$$R(\widetilde{\alpha_1}) - r + \widetilde{\alpha_1} R'(\widetilde{\alpha_1}) = -\mu(\beta\pi^* + \alpha_1(1-\beta)\pi^* + (1-\alpha_1)k\pi^*)$$

The solution of the equation set will be an equilibrium in the market. As the two banks are identical and the first order conditions are symmetric, we can assume that there exist α_1^{4*} satisfying the equation $R(\alpha_1^{4*}) - r + \alpha_1^{4*}R'(\alpha_1^{4*}) = -\mu(\beta\pi^* + \alpha_1^{4*}(1-\beta)\pi^* + (1-\alpha_1^{4*})k\pi^*)$. Obviously, $\alpha_1 = \widetilde{\alpha_1} = \alpha_1^{4*}$ is an equilibrium in the market.

The significance of this result is that for two identical banks there is a possibility of equilibrium while two banks choose to invest on the independent assets with the same risk. In practice, the expected returns are different if two banks invest on assets with different risks. The bank receiving lower returns will not tolerate such a consequence. Then the most possible equilibrium is that the two banks invest on independent assets with the same risk and amount of return. We denote the expected profits of the respective bank as:

$$\pi^{4*} = \alpha_1^{4*}(R(\alpha_1^{4*}) - r) + \mu[\alpha_1^{4*}(\alpha_1^{4*}\pi^* + (1-\alpha_1^{4*})(1+k)\pi^*) + (1-\alpha_1^{4*})(\alpha_1^{4*} \times 0 \times \pi^* + (1-\alpha_1^{4*})(1-\beta)\pi^*)]$$

The α_1^{4*} can be expressed as $\alpha_1^{4*} = \alpha_1^{4*}(\beta)$ incorporating the equation $R(\alpha_1^{4*}) - r + \alpha_1^{4*}R^{'}(\alpha_1^{4*}) = -\mu(\beta\pi^* + \alpha_1^{4*}(1-\beta)\pi^* + (1-\alpha_1^{4*})k\pi^*)$. And $\alpha_1^{4*}(\beta) > \alpha_1^{2*}(\beta)$. From this equation, we get the derivative of $\alpha_1^{4*}(\beta)$:

$$\alpha_1^{4*'}(\beta) = \frac{-\mu\left(1-\alpha_1^{4*}(\beta)\right)\pi^*}{R^{'}\left(\alpha_1^{4*}(\beta)\right) + R^{'}\left(\alpha_1^{4*}(\beta)\right) + \alpha_1^{4*}(\beta)R^{''}\left(\alpha_1^{4*}(\beta)\right) + \mu(1-\beta-k)\pi^*}$$

$\alpha_1^{4*'}(\beta) > 0$. (See proof 7 in Appendix)

Similarly, π^{4*} is also the function of β, $\pi^{4*} = \pi^{4*}(\beta)$. $\pi^{4*'}(\beta) = \mu[-\alpha_1^{4*}k\pi^* - (1-\alpha_1^{4*})(1-$ $\beta\pi*\alpha14*'\beta$>0, which indicates $\pi4*$ is increasing in β. When β<1−k, $\pi4*\beta$<$\pi2*\beta$. (See proof 8 in Appendix)

Lemma 5 When banks choose to diversify their investments, as the equity share of the regulator increases each bank lowers investment risk, and systemic risk is also reduced. In the meantime, the expected profits will be reduced.

In the case of diversified investment, the banking aggregate outputs are

$$\Pi(\beta) = 2\alpha_1^{4*}(R(\alpha_1^{4*}) - r) + 2\mu\pi^*$$

$\Pi(\beta)$ is decreasing in β (see proof 9 in Appendix). This conclusion once again indicates that the more rigorous management the regulator enforces on banks, the less expected profits will be generated in the banking sector.

In this case, the social welfare is

$$W(\beta) = 2\pi^{4*} + 2\alpha_1^{4*}(\beta)(1-\alpha_1^{4*}(\beta))(p-r) + 2\left(1-\alpha_1^{4*}(\beta)\right)^2(\mu\beta\pi^* - r)$$

For simplicity, the equation can be rewritten as

$W(\beta) = 2\alpha_1^{4*}(R(\alpha_1^{4*}) - r) + 2\mu\pi^* - 2\left(1-\alpha_1^{4*}(\beta)\right)$.And$W^{'}(\beta) = 2\alpha_1^{4*'}(\beta)R(\alpha_1^{4*})\left(1 - E_{R\alpha}\left(\alpha_1^{4*}(\beta)\right)\right)$.

Theoretically, $\exists\bar{\bar{\beta}}$, $\bar{\bar{\beta}} < \bar{\beta}$, if $\beta < \bar{\bar{\beta}}$, $W^{'}(\beta) < 0$, the social welfare increases; if $\beta > \bar{\bar{\beta}}$, $W^{'}(\beta) < 0$, the social welfare decreases. If $\beta = \bar{\bar{\beta}}$, $W^{'}(\beta) = 0$, the social welfare is maximized (see proof 10 in Appendix).

Lemma 6 If the banks choose diversified investments:

(1) The banking aggregate outputs are decreasing in β.

(2) The variation tendency of the social welfare in β is:

i. If $\bar{\bar{\beta}} < 1$, when $\beta < \bar{\bar{\beta}}$, the social welfare is increasing in β; when $\beta > \bar{\bar{\beta}}$, the social welfare is decreasing in β; when $\beta = \bar{\bar{\beta}}$ the social welfare is maximized.

ii. If $\bar{\bar{\beta}} \geq 1$, the social welfare is increasing in β all the time.

4.4 The regulator's strategy at t=0

The regulator intends to make policy to maximize the social welfare and the outputs of the banking sector and to minimize the investment risk to avoid systemic risk. Through the analysis above, these three targets are impossible to achieve simultaneously. We term this phenomenon objective realization-inconsistency. The regulator's strategy at $t = 1$ is decided by the impact of banks' behavior not being completely controlled by the regulator and this is common knowledge to the regulator and banks. But the setting of β is not decided by the second period results. The regulator can set β freely at the beginning of the first period to restrain banks' behavior.

The regulator's objective vector (W, Π, Ri) is actually a function of β. The regulator should balance the three targets to set β. β represents the penalty when a bank is in default. The larger β is, the more severe the penalty is. The severe penalty helps to reduce risk, but also circumscribes the activities of banks, reducing outputs. The change of total social welfare level is a complicated process, creating difficulties for the formulation of bailout policy.

In terms of the risk, when $\beta > \beta^*$, the bank deviates from parallel investing. $\alpha_1^{2*}(\beta^{'}) < \alpha_1^{2*}(\beta) < \alpha_1^{4*}(\beta)$, $\beta^{'} \leq \beta^* < \beta$. That suggests that the risk of an individual bank when investing in independent assets is absolutely lower than when parallelly investing. Meanwhile, $1 - \alpha_1^{2*}(\beta^{'}) > \left(1-\alpha_1^{4*}(\beta)\right)^2$. This implies that the systemic risk is absolutely reduced through diversified investment. Additionally, whether in the case of diversified investment or parallel investing, the risks of an individual bank and the system are decreasing in β.

Proposition 3 With the β exceeding the critical value β^*, the banks begin to diversify the investment instead of parallell invest. The risk of an individual bank and the systemic risk when investing in independent assets are absolutely lower than those when parallelly investing. Meanwhile, in both cases, the risk of an individual bank and the systemic risk are decreasing in β.

In terms of banking outputs, in the case of parallel investing, $\Pi(\beta') = 2\alpha_1^{2*}(\beta')\left(R\left(\alpha_1^{2*}(\beta')\right) - r\right) + 2\mu\pi^*$; in the case of diversified investment $\Pi(\beta) = 2\alpha_1^{4*}\left(R\left(\alpha_1^{4*}(\beta)\right) - r\right) + 2\mu\pi^*, \beta' \leq \beta^* < \beta$. $\alpha^* < \alpha_1^{2*}(\beta') < \alpha_1^{4*}(\beta)$, $\alpha_1^{2*}(\beta')\left(R\left(\alpha_1^{2*}(\beta')\right) - r\right) > \alpha_1^{4*}\left(R\left(\alpha_1^{4*}(\beta)\right) - r\right)$. So the banking outputs in the case of parallel investing are absolutely larger than those of diversified investment. In addition, the outputs of the two cases are decreasing in β.

Proposition 4 The aggregate outputs of the banking sector are decreasing in β. In particular, the outputs in the case of parallel investing are more than those in the case of diversified investment.

The variation of the social welfare situation is more complicated because it includes the costs caused by provision of funds. In the case of parallel investing, the social welfare is $W(\beta') = 2\alpha_1^{2*}(\beta')\left(R\left(\alpha_1^{2*}(\beta')\right) -\right.$ *r+2μπ∗−21−α12∗β′r*;

In the case of diversified investment, the social welfare is: $W(\beta) = 2\alpha_1^{4*}(\beta)\left(R\left(\alpha_1^{4*}(\beta)\right) - r\right) + 2\mu\pi^* - 2\left(1 - \alpha_1^{4*}(\beta)\right)r$.

(1) $\bar{\bar{\beta}} < \bar{\beta} \leq \beta^*$

If the regulator set $\beta = \bar{\beta}$, the banks prefer to parallelly invest and the social welfare is maximized. If the regulator wants to lower risks and force banks to invest in independent assets, the regulator should set β marginally higher than β^* ($\beta = \beta^* + \Delta\beta$, $\Delta\beta \to 0$) to maintain the social welfare on its maximum in this case. And $W(\bar{\beta}) > W(\beta^* + \Delta\beta)$ (see proof 11 in Appendix). The results above can be demonstrated by Figure 2:

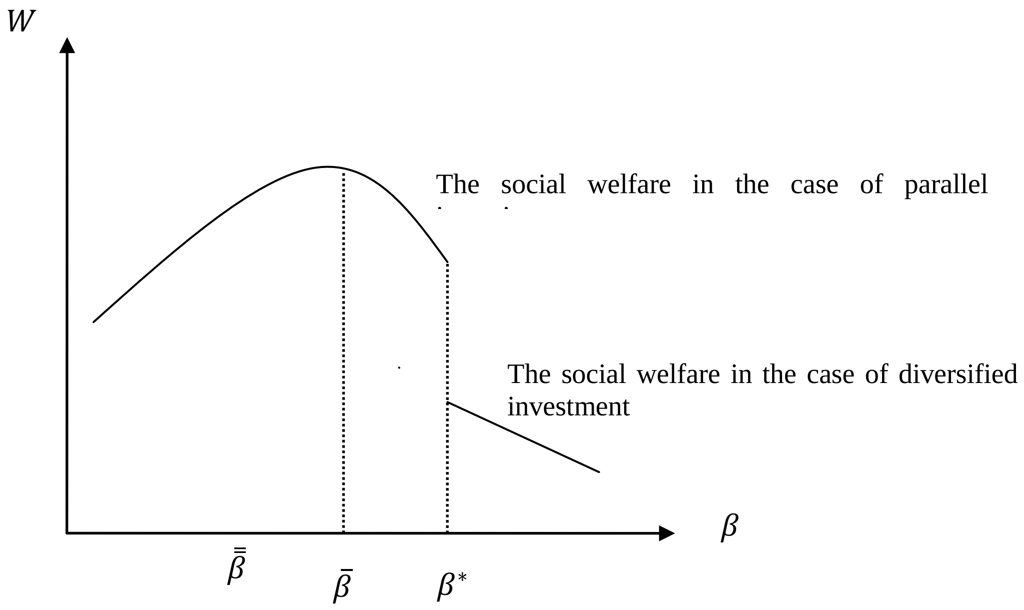

Figure 2

(2) $\bar{\bar{\beta}} \leq \beta^* < \bar{\beta}$

If the regulator set $\beta = \beta^*$, the banks prefer to parallelly invest and the social welfare is maximized. If the regulator wants to lower risks and force banks to invest in independent assets, the regulator should set β slightly higher than β^* ($\beta = \beta^* + \Delta\beta$, $\Delta\beta \to 0$) to maintain the social welfare on its maximum. in this case $W(\beta^*)$ may be larger, smaller or equal to $W(\beta^* + \Delta\beta)$ (see proof 12 in Appendix). The results, above, can be demonstrated by Figure 3:

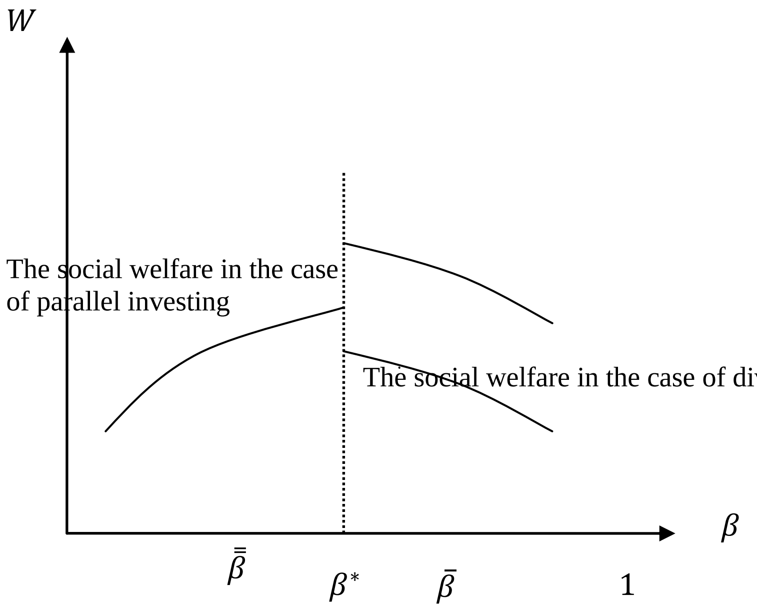

Figure 3

(3) $\beta^* < \bar{\bar{\beta}} < \bar{\beta}$

If the regulator set $\beta = \beta^*$, the banks prefer to parallelly invest and the social welfare is maximized. If the regulator set $\beta = \bar{\bar{\beta}}$, the banks prefer diversified investment and the social welfare is maximized. $W(\beta^*) < W\left(\bar{\bar{\beta}}\right)$ (see proof 13 in Appendix). The above results are demonstrated in Figure 4:

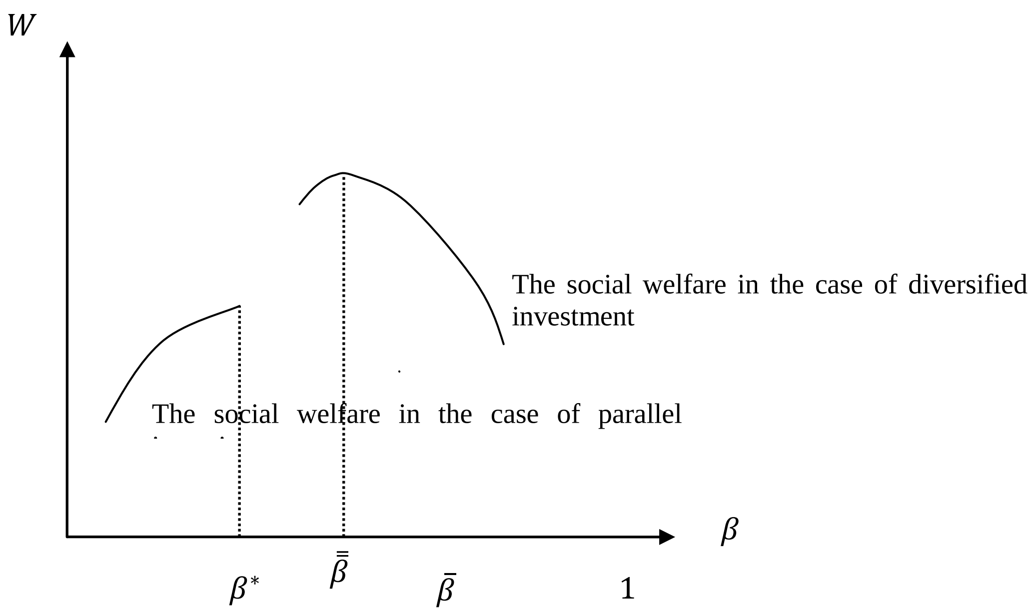

Figure 4

Lemma 7 For social welfare maximization, the regulator should set β:

i. $\bar{\bar{\beta}} < \bar{\beta} \leq \beta^*$, the regulator sets $\beta = \bar{\beta}$, the social welfare is maximized in the case of parallel investing; or $\beta = \beta^* + \Delta\beta$, $\Delta\beta \to 0$, the social welfare is maximized in the case of diversified investment. $W\left(\bar{\beta}\right) > W(\beta^* + \Delta\beta)$.

ii. $\bar{\bar{\beta}} \leq \beta^* < \bar{\beta}$, the regulator sets $\beta = \beta^*$, the social welfare is maximized in the case of parallel investing; or $\beta = \beta^* + \Delta\beta$, $\Delta\beta \to 0$, the social welfare is maximized in the case of diversified investment. $W(\beta^*)$ may be larger, smaller or equal to $W(\beta^* + \Delta\beta)$.

iii. $\beta^* < \bar{\bar{\beta}} < \bar{\beta}$, the regulator sets $\beta = \beta^*$, the social welfare is maximized in the case of parallel investing; or $\beta = \bar{\bar{\beta}}$, the social welfare is maximized in the case of diversified investment. $W(\beta^*) < W\left(\bar{\bar{\beta}}\right)$.

Proposition 5 The variation tendency of three components of the regulator's objective vector in β is not consistent, which is called objective realization-inconsistency. The regulator cannot pursue a perfect bailout policy to optimize all the three targets at the same time, but can operationalize a relatively effective policy to balance the three targets.

(1) Risk aversion-oriented: the policy focusing on lowering risks:

i. If $\bar{\bar{\beta}} < \bar{\beta} \leq \beta^*$ or $\bar{\bar{\beta}} \leq \beta^* < \bar{\beta}$, the regulator sets $\beta = \beta^* + \Delta\beta$, $\Delta\beta \rightarrow 0$.

ii. If $\beta^* < \bar{\bar{\beta}} < \bar{\beta}$, the regulator sets $\beta = \bar{\bar{\beta}}$ ($\bar{\bar{\beta}}$ is feasible).[6]

The regulator can force banks to diversify their investments, lowering individual bank's risk and the systemic risk by a risk aversion-oriented policy. Social welfare is also maximized in this case. Increasing β, the regulator can lower risk further, but the social welfare and the banking outputs are also decreased.

(2) Output-oriented: the policy focusing on increasing banking outputs:

i. If $\bar{\bar{\beta}} < \bar{\beta} \leq \beta^*$, the regulator sets $\beta = \bar{\beta}$.

ii. If $\bar{\bar{\beta}} \leq \beta^* < \bar{\beta}$ or $\beta^* < \bar{\bar{\beta}} < \bar{\beta}$, the regulator sets $\beta = \beta^*$.

With output-oriented policy, banks prefer to parallelly invest, and the banking outputs are maintained at a high level. The social welfare is maximized in this case. Decreasing β, the regulator can increase banking outputs, but the social welfare is decreased while the individual bank's risk and the systemic risk are increased.

5. Financial Innovation and Bailout Policies

In order to pursue more profits, lower risks, and meet market needs for financial services, as well as to better adapt to the environment, the financial sector constantly seeks to innovate. Silber (1983) argues that multiple endogenous and exogenous constraints on the activities of the financial system and its actors prevent the pursuit of more profits and that the majority of financial instruments or practices are innovations oriented to attenuate such constraints in order to increase profits.

According to the analysis above, it can be noted that to restrain banks' behavior and lower systemic risk the regulator is required to balance the investment risk and social welfare in order to find a suitable way to supervise the development of the banking industry. Financial derivatives are recognized as the most important financial innovation tools since the 1990s: they help to transfer banking risk to other investors and lower the probability of default, thus bring higher long-term profits. The following analysis uses a particular type of representative financial derivative, credit-default swaps, and compares it with the previous analyses and provides a discussion regarding whether financial derivatives have an active role in bank regulation.

We continue the analysis using the two banks model introduced previously. The difference is that after introducing credit-default swaps, banks can purchase credit protection in the derivative market. Whenever the banks suffer losses from investment, the sellers promise to compensate the banks for their corresponding losses. According to the previous analysis we assume that the bank needs to purchase protection for 1 unit of wealth, and the price is p_r. In the completely competitive credit market, the profit of the seller is 0, so according to the bank's investment, $p_r = (1-\alpha)R(\alpha)$.[7] Out of the Consideration of profit maximization, the bank will not purchase credit protection in the second period (see proof 14 in Appendix). In this case, a bank's objective function is

$$\max R(\alpha_1) - r - p_r + \mu\pi^*$$

Since $p_r = (1-\alpha_1)R(\alpha_1)$, the function can be expresses as:

$$\max \alpha_1 R(\alpha_1) - r + \mu\pi^*$$

The first-order condition of maximization is

$$R(\alpha_1) + \alpha_1 R^{'}(\alpha_1) = 0$$

The solution of the equation is $\alpha_1 = \alpha_1^{5*}, \alpha_1^{5*} > \alpha^*$. Let $\pi^{5*} = \alpha_1^{5*} R(\alpha_1^{5*}) - r + \mu\pi^*$. Furthermore (see proof 15 in Appendix)

we can prove that: $\alpha_1^{2*}(0) = \alpha^* < \alpha_1^{5*}$, $\pi^{2*}(0) - \pi^{5*} > 0$.

i. $r - \mu\pi^* < 0$, $\alpha_1^{2*}(1) > \alpha_1^{5*}$, $\pi^{2*}(1) - \pi^{5*} < 0$.

ii. $r - \mu\pi^* \geq 0$, $\alpha_1^{2*}(1) \leq \alpha_1^{5*}$, $\pi^{2*}(1) - \pi^{5*} \geq 0$.

Accordingly, we find that if $r - \mu\pi^* \geq 0$, no matter how many shares the regulator obtained after bailout, the expected profits achieved by purchasing credit protection are no more than that by parallel investing.

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[6] "$\bar{\bar{\beta}}$ is feasible" indicates $\bar{\bar{\beta}} \leq 1$. Furthermore, the moral hazard of bank owners may be a problem in β setting, which is not discussed in this paper. See Hart and Moore (1994) and Acharya and Yorulmazer (2007).

[7] Duffee and Zhou(2001) determine a pricing equation based on 0 profit assumption when analyzing the protection purchase for a part of the bank's assets. We assume the bank has 1 unit of wealth. The pricing for this 1 unit of wealth is similar to their analysis.

So, the credit protection cannot replace parallel investing for the risk neutral bank. If $r-\mu\pi^* < 0$, there exists β^{**}, so that the profits achieved both by purchasing credit protection and by parallel investing are equal. Additionally, if $\beta^* > \beta^{**}$, it is possibly better for the bank to choose credit protection than parallel investing.

Lemma 8 When $r-\mu\pi^* < 0$ then $\exists\beta^{**}$, if $\beta \le \beta^{**}$, $\pi^{2*}(\beta) \ge \pi^{5*}$; If $\beta > \beta^{**}$, $\pi^{2*}(\beta) < \pi^{5*}$.

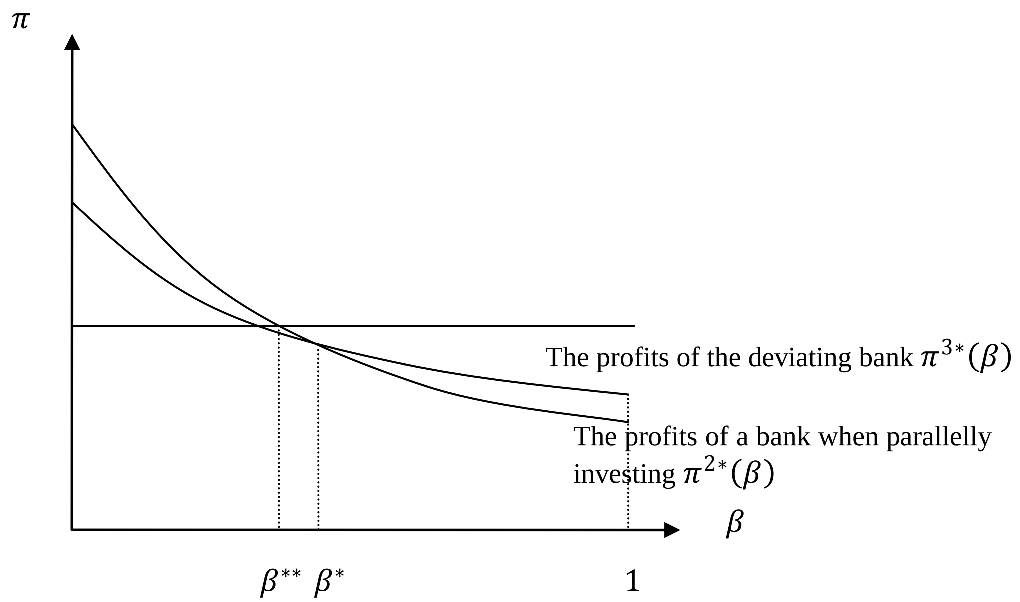

Figure 5

It can be seen from Figure 5 that when $r-\mu\pi^* < 0$, if $\beta^* > \beta^{**}$, banks will lose interest in parallel investment and purchase credit protection as long as the regulator makes $\beta > \beta^{**}$.

In terms of the risk, credit protection can ensure banks receive a certain amount of profit, which acts to disperse risks. Actually, there is no risk for the bank; and as a result there exists no system risk.

In terms of the aggregate outputs of banking, purchasing credit protection can also help maintain the banks' aggregate outputs at a high level. Compared to the above results, when $\beta^* > \beta > \beta^{**}$, the banking aggregate outputs are higher than the case of parallel investing when $\beta = \beta^*$, and significantly higher than the aggregate outputs of diversified investment without credit protection.

In terms of social welfare, because there is no risk in bank investment the regulator does not need to bailout the banks. Consequently, there is no loss of wealth and the social welfare outputs are equal to the banking aggregate outputs.

Compared with the risk aversion-oriented bailout policy, credit protection results in a lowering of the risks to an individual bank and the system whilst also increasing banks' aggregate outputs. The social welfare is equal to the banking aggregate outputs and is higher than the aggregate outputs associated with diversified investment without credit protection. Therefore, credit protection is better than the risk aversion-oriented bailout policy. Compared with the output-oriented bailout policy, credit protection leads to a large decrease in risks. Although the outputs and social welfare may not increase, they remain at a high level relative to that associated with parallel investment.

Therefore under certain conditions credit protection can play a positive role in the regulation of all banks.

Proposition 6 When $r-\mu\pi^* < 0$ and $\beta^{**} < \beta^*$, credit default swap (understood as a representative of financial derivatives) plays a positive role in banking regulation. It not only significantly reduces risk but also maintains high levels of banking outputs and social welfare.

6. Conclusion

Due to the collective behavior of banks, bank regulation is increasingly difficult. This research has focused attention on the regulation, the behavior of individual banks and the behavior of the banking system. Individual banks are normally the recipients of regulation and an individual bank's strategy cannot easily influence regulation strategy. In contrast, the collective behavior of banks exerts significant influence on the regulator. We argue that it is extremely important to analyze the regulatory strategy directed at the collective behavior of banks.

The purpose of this paper is to analyze the optimal strategy for the regulator *vis-a-vis* the whole banking system. This paper does three things. First, we have provided an extension of herding theory. We argue that parallel investing is one investment strategy that banks seeking profit maximization would choose. It becomes easier for banks to search for the regulator's guarantee when in trouble through parallel investing. Banks not only offer loans to the same industries, but also achieve high correlation in terms of other asset operations. As a matter of fact, we can regard parallel investing as a more extreme form of clustered investment. Second, we provide a discussion regarding the formation of an optimal strategy; in particular an optimal bailout based on parallel investing. We argue that the regulator's objective is not simply the maximization of social welfare, but a three dimensional vector. The inconsistent variation of each component of this vector renders regulators incapable of finding a perfect strategy that can achieve all three targets simultaneously; a phenomenon we term objective realization-inconsistency. However, a relatively effective bailout strategy capable of balancing the three aspects of the objective can be identified. Finally, we have evaluated the contribution of financial derivatives on reducing banking risk and increasing outputs. Here, the conclusion is that financial derivatives can play a positive role in the regulation of all banks.

Appendix

1.

$g(\alpha) = R(\alpha) - r + \alpha R'(\alpha)$, $g'(\alpha) = R'(\alpha) + R'(\alpha) + \alpha R''(\alpha)$.

$\because R'(\alpha) < 0,\ R''(\alpha) < 0.\ \therefore\ g'(\alpha) < 0$. Assume that $\exists \alpha^* \in (0,1)$, $R(\alpha^*) - r + \alpha^* R'(\alpha^*) = 0$. Then if $\alpha > \alpha^*$, $R(\alpha^*) - r + \alpha^* R'(\alpha^*) < 0$, $\alpha(R(\alpha) - r)$ is decreasing in α; if $\alpha < \alpha^*$, $R(\alpha^*) - r + \alpha^* R'(\alpha^*) > 0$, $\alpha(R(\alpha) - r)$ is increasing in α.

2.

Two first order conditions:

$$R(\alpha^*) - r + \alpha^* R'(\alpha^*) = 0$$
$$\sigma R(\alpha^\#) - r + \alpha^\# \sigma R'(\alpha^\#) = 0$$

Rewrite the two equations:

$$R(\alpha^*) + \alpha^* R'(\alpha^*) = r$$
$$R(\alpha^\#) + \alpha^\# R'(\alpha^\#) = \frac{r}{\sigma}$$

Let $t(\alpha) = R(\alpha) + \alpha R'(\alpha)$. $t'(\alpha) = R'(\alpha) + R'(\alpha) + \alpha R''(\alpha) < 0$. In addition, $\frac{r}{\sigma} > r$, $\therefore \alpha^* > \alpha^\#$.

Consider π^* and $\pi^\#$:

$$\pi^* = \alpha^* (R(\alpha^*) - r)$$
$$\pi^\# = \alpha^\# (\sigma R(\alpha^\#) - r)$$

From the first order conditions we know:

$$R(\alpha^*) - r = -\alpha^* R'(\alpha^*)$$
$$\sigma R(\alpha^\#) - r = -\alpha^\# \sigma R'(\alpha^\#)$$

$\because \alpha^* > \alpha^\#, R'' < 0,\ R'(\alpha^*) < R'(\alpha^\#) < 0$, then $\alpha^* R'(\alpha^*) < \alpha^\# R'(\alpha^\#) < 0$, $-\alpha^* R'(\alpha^*) > -\alpha^\# R'(\alpha^\#) > 0$, $\therefore -\alpha^* R'(\alpha^*) > -\sigma\alpha^\# R'(\alpha^\#) > 0$, that is $R(\alpha^*) - r > \sigma R(\alpha^\#) - r > 0$, $\therefore\ \alpha^* (R(\alpha^*) - r) > \alpha^\# (\sigma R(\alpha^\#) - r) > 0, \pi^* > \pi^\#$.

3.

According to the first order condition:

$$[R'(\alpha_1^{2*}) + R'(\alpha_1^{2*}) + \alpha_1^{2*} R''(\alpha_1^{2*})]\alpha_1^{2*'}(\beta) = -\mu\pi^*$$

That is

$$\alpha_1^{2*'}(\beta) = \frac{-\mu\pi^*}{R'(\alpha_1^{2*}) + R'(\alpha_1^{2*}) + \alpha_1^{2*} R''(\alpha_1^{2*})}$$

$\alpha_1^{2*'}(\beta) > 0$, α_1^{2*} is increasing in β, so the risk is decreasing in β.

The maximization of profit is $\pi^{2*} = \alpha_1^{2*}(R(\alpha_1^{2*}) - r) + \mu(\alpha_1^{2*}\pi^* + (1 - \alpha_1^{2*})(1 - \beta)\pi^*)$.

$\frac{\partial \pi^{2*}}{\partial \beta} = (\alpha_1^{2*}(R(\alpha_1^{2*}) - r) + \mu(\alpha_1^{2*}\pi^* + (1 - \alpha_1^{2*})(1 - \beta)\pi^*))'_{\alpha_1^{2*}} \times \alpha_1^{2*'}(\beta) - \mu(1 - \alpha_1^{2*})\pi^* = 0 - \mu(1 - \alpha_1^{2*})\pi^* < 0$. So in the case of parallel investing, the maximization of bank's profit π^{2*} is decreasing in β.

4.

$$\Pi(\beta) = 2\alpha_1^{2*}(\beta)\left(R\left(\alpha_1^{2*}(\beta)\right) - r\right) + 2\mu\pi^*$$

$$\Pi^{'}(\beta) = 2\left(R\left(\alpha_1^{2*}(\beta)\right) - r + \alpha_1^{2*}(\beta)R^{'}\left(\alpha_1^{2*}(\beta)\right)\right)\alpha_1^{2*^{'}}(\beta)$$

$R\left(\alpha_1^{2*}(\beta)\right) - r + \alpha_1^{2*}(\beta)R^{'}\left(\alpha_1^{2*}(\beta)\right) = -\mu\beta\pi^*$, $\therefore$ $\Pi^{'}(\beta) < 0$. So $\Pi(\beta)$ is decreasing inβ.

5.

$E_{R\alpha} = -\frac{\alpha}{R}R^{'}(\alpha)$,$E_{R\alpha}\left(\alpha_1^{2*}(\beta)\right) = -\frac{\alpha_1^{2*}(\beta)}{R\left(\alpha_1^{2*}(\beta)\right)}R^{'}\left(\alpha_1^{2*}(\beta)\right)$.

$$\frac{\mathrm{d}E_{R\alpha}\left(\alpha_1^{2*}(\beta)\right)}{\mathrm{d}\beta} = -\frac{\left(R^{'}\left(\alpha_1^{2*}(\beta)\right)+\alpha_1^{2*}(\beta)R^{''}\left(\alpha_1^{2*}(\beta)\right)\right)R\left(\alpha_1^{2*}(\beta)\right)-\alpha_1^{2*}(\beta)\left(R^{'}\left(\alpha_1^{2*}(\beta)\right)\right)^2}{R^2\left(\alpha_1^{2*}(\beta)\right)}\alpha_1^{2*^{'}}(\beta) > 0$$

So $E_{R\alpha}\left(\alpha_1^{2*}(\beta)\right)$ is increasing inβ.

Meanwhile, $W^{'}(\beta) = 2(-\mu\beta\pi^* + r)\alpha_1^{2*^{'}}$. So, numerically, when $\beta = \frac{r}{\mu\pi^*}$,$W^{'}(\beta) = 0$. $W^{'}(\beta) = 2\alpha_1^{2*^{'}}(\beta)R\left(\alpha_1^{2*}(\beta)\right)\left(1 - E_{R\alpha}\left(\alpha_1^{2*}(\beta)\right)\right)$, if$\exists\beta = \bar{\beta}$, $W^{'}(\beta) = 0$, then $E_{R\alpha}\left(\alpha_1^{2*}(\bar{\beta})\right) = 1$. $\because$ $E_{R\alpha}\left(\alpha_1^{2*}(\beta)\right)$ is increasing in β, then $\bar{\beta}$ is unique. $\because$when $\beta = \frac{r}{\mu\pi^*}$, $W^{'}(\beta) = 0$, $\therefore$ $\bar{\beta} = \frac{r}{\mu\pi^*}$. If $\beta < \bar{\beta}$, $W^{'}(\beta) > 0$, the social welfare is increasing in β; if $\beta > \bar{\beta}$, $W^{'}(\beta) < 0$, the welfare is decreasing in β.

6.

$$\pi^{3*} = \alpha_1^{3*}(R(\alpha_1^{3*}) - r) + \mu[\alpha_1^{3*}(\pi^* + (1 - \alpha_1^{2*})k\pi^*) + (1 - \alpha_1^{3*})(1 - \beta)\pi^* - (1 - \alpha_1^{3*})\alpha_1^{2*}(1 - \beta)\pi^*]$$
$$= \alpha_1^{3*}(R(\alpha_1^{3*}) - r) + \mu[\alpha_1^{3*}\pi^* + (1 - \alpha_1^{3*})(1 - \beta)\pi^* + \alpha_1^{3*}(1 - \alpha_1^{2*})k\pi^* - (1 - \alpha_1^{3*})\alpha_1^{2*}(1 - \beta)\pi^*]$$
$$= \alpha_1^{3*}(R(\alpha_1^{3*}) - r) + \mu[\alpha_1^{3*}\pi^* + (1 - \alpha_1^{3*})(1 - \beta)\pi^*] + \mu[\alpha_1^{3*}(1 - \alpha_1^{2*})k\pi^* - (1 - \alpha_1^{3*})\alpha_1^{2*}(1 - \beta)\pi^*].$$
$$\pi^{2*} = \alpha_1^{2*}(R(\alpha_1^{2*}) - r) + \mu(\alpha_1^{2*}\pi^* + (1 - \alpha_1^{2*})(1 - \beta)\pi^*).$$

Let$g(\alpha) = \alpha(R(\alpha) - r) + \mu[\alpha\pi^* + (1 - \alpha)(1 - \beta)\pi^*]$,

$$\pi^{2*} - \pi^{3*} = g(\alpha_1^{2*}) - g(\alpha_1^{3*}) - \mu[\alpha_1^{3*}(1 - \alpha_1^{2*})k\pi^* - (1 - \alpha_1^{3*})\alpha_1^{2*}(1 - \beta)\pi^*]$$
$$= g^{'}(\overline{\alpha_1})(\alpha_1^{2*} - \alpha_1^{3*}) - \mu[\alpha_1^{3*}(1 - \alpha_1^{2*})k\pi^* - (1 - \alpha_1^{3*})\alpha_1^{2*}(1 - \beta)\pi^*],\ \alpha_1^{2*} < \overline{\alpha_1} < \alpha_1^{3*}.$$

$g^{'}(\overline{\alpha_1}) = R(\overline{\alpha_1}) - r + \overline{\alpha_1}R^{'}(\overline{\alpha_1}) + \mu\beta\pi^*$. $R(\alpha_1^{2*}) - r + \alpha_1^{2*}R^{'}(\alpha_1^{2*}) + \mu\beta\pi^* = 0$, $\alpha_1^{2*} < \overline{\alpha_1}$.

$\therefore$ $R(\alpha_1^{2*}) - r + \alpha_1^{2*}R^{'}(\alpha_1^{2*}) > R(\overline{\alpha_1}) - r + \overline{\alpha_1}R^{'}(\overline{\alpha_1})$, then $R(\overline{\alpha_1}) - r + \overline{\alpha_1}R^{'}(\overline{\alpha_1}) + \mu\beta\pi^* < 0$,

$g^{'}(\overline{\alpha_1})(\alpha_1^{2*} - \alpha_1^{3*}) > 0$.

Let$-\mu[\alpha_1^{3*}(1 - \alpha_1^{2*})k\pi^* - (1 - \alpha_1^{3*})\alpha_1^{2*}(1 - \beta)\pi^*] \geq 0$, that is$(1 - \alpha_1^{3*})\alpha_1^{2*}(1 - \beta)\pi^* \geq \alpha_1^{3*}(1 -$ *α12∗kπ∗*, then *1−β≥α13∗1−α12∗k1−α13∗α12∗*, *β≤1−α13∗1−α12∗k1−α13∗α12∗*.

So if $\beta \leq 1 - \frac{\alpha_1^{3*}(1-\alpha_1^{2*})k}{(1-\alpha_1^{3*})\alpha_1^{2*}}$, $\pi^{2*} > \pi^{3*}$. The value of $\frac{\alpha_1^{3*}(1-\alpha_1^{2*})k}{(1-\alpha_1^{3*})\alpha_1^{2*}}$ is relevant to the definite form of the return function and the efficiency of the outsider.

According to the equation$R(\alpha_1^{3*}) - r + \alpha_1^{3*}R^{'}(\alpha_1^{3*}) = -\mu(\beta\pi^* + \alpha_1^{2*}(1 - \beta)\pi^* + (1 - \alpha_1^{2*})k\pi^*)$, $\alpha_1^{3*} = \alpha_1^{3*}(\beta)$.

$$(R^{'}(\alpha_1^{3*}) + R^{'}(\alpha_1^{3*}) + \alpha_1^{3*}R^{''}(\alpha_1^{3*}))\alpha_1^{3*^{'}}(\beta) = -\mu(1 - \alpha_1^{2*})\pi^*$$

$$\alpha_1^{3*^{'}}(\beta) = \frac{-\mu(1 - \alpha_1^{2*})\pi^*}{R^{'}(\alpha_1^{3*}) + R^{'}(\alpha_1^{3*}) + \alpha_1^{3*}R^{''}(\alpha_1^{3*})}$$

$\frac{\partial\pi^{3*}}{\partial\beta} =$

$(\alpha_1^{3*}(R(\alpha_1^{3*}) - r) + \mu[\alpha_1^{3*}(\alpha_1^{2*}\pi^* + (1 - \alpha_1^{2*})(1 + k)\pi^*) + (1 - \alpha_1^{3*})(1 - \alpha_1^{2*})(1 - \beta)\pi^*])^{'}_{\alpha_1^{3*}}\alpha_1^{3*^{'}}(\beta) - (1 - \alpha_1^{3*})(1 - \alpha_1^{2*})\pi^* = 0 - (1 - \alpha_1^{3*})(1 - \alpha_1^{2*})\pi^* < 0$

So in the case of parallel investing, the bank's profits are decreasing in β.

Compare $\frac{\partial\pi^{2*}}{\partial\beta}$ with $\frac{\partial\pi^{3*}}{\partial\beta}$, $\frac{\partial^2\pi^{2*}}{\partial\beta^2}=\mu\alpha_1^{2*'}(\beta)\pi^*>0$, $\frac{\partial^2\pi^{3*}}{\partial\beta^2}=\mu\alpha_1^{3*'}(\beta)\left(1-\alpha_1^{2*}(\beta)\right)\pi^*+\mu\alpha_1^{2*'}(\beta)\left(1-\right.$ *α13∗βπ∗>0*, and*∂π2∗∂β=−μ1−α12∗βπ∗<−μ1−α13∗β1−α12∗βπ∗=∂π3∗∂β*.

If $\beta=0$, the first order condition of parallel investing is

$$R(\alpha_1)-r+\alpha_1R'(\alpha_1)=0$$

The solution $\alpha_1=\alpha_1^{2*}=\alpha^*, \pi^{2*}=(1+\mu)\pi^*$.

The first order condition of deviation is

$$R(\alpha_1)-r+\alpha_1R'(\alpha_1)=-\mu(\alpha_1^{2*}\pi^*+(1-\alpha_1^{2*})k\pi^*)$$

$\alpha_1=\alpha_1^{3*},\pi^{3*}=\alpha_1^{3*}(R(\alpha_1^{3*})-r)+\mu[\alpha_1^{3*}(\alpha_1^{2*}\pi^*+(1-\alpha_1^{2*})(1+k)\pi^*)+(1-\alpha_1^{3*})(1-\alpha_1^{2*})\pi^*]$

When $\beta\le 1-\frac{\alpha_1^{3*}(1-\alpha_1^{2*})k}{(1-\alpha_1^{3*})\alpha_1^{2*}}$, $\pi^{2*}>\pi^{3*}$. We assume that the regulator will not set the price of failed banks' assets at a low level for the rapid replenishment of funds, so k is not very large. If $\beta=0$, $\alpha_1^{2*}(0)=\alpha^*$, $\frac{\alpha_1^{3*}(1-\alpha_1^{2*})k}{(1-\alpha_1^{3*})\alpha_1^{2*}}=\frac{\alpha_1^{3*}(1-\alpha^*)k}{(1-\alpha_1^{3*})\alpha^*}$ is increasing in α_1^{3*}. Meanwhile $\alpha_1^{3*}=\alpha_1^{3*}(0)$ is the minimization, so we assume that $\beta=0\le 1-\frac{\alpha_1^{3*}(1-\alpha^*)k}{(1-\alpha_1^{3*})\alpha^*}$. In the real economy, this assumption indicates that if β is small enough, the banks have incentive to parallelly invest and take higher risk to seek guarantee. Meanwhile, it also indicates that because the regulator dose not set the price at a low level, i.e. k is not large enough, the parallel investing is more profitable when β is smaller. As a result, $\pi^{2*}(0)>\pi^{3*}(0)$.

If $\beta=1$, the first order condition in the case of parallel investing is

$$R(\alpha_1)-r+\alpha_1R'(\alpha_1)=-\mu\pi^*$$

The solution $\alpha_1=\alpha_1^{2*}, \pi^{2*}=\alpha_1^{2*}(R(\alpha_1^{2*})-r)+\mu\alpha_1^{2*}\pi^*$

The first order condition of deviation is

$$R(\alpha_1)-r+\alpha_1R'(\alpha_1)=-\mu(\pi^*+(1-\alpha_1^{2*})k\pi^*)$$

The solution$\alpha_1=\alpha_1^{3*}, \pi^{3*}=\alpha_1^{3*}(R(\alpha_1^{3*})-r)+\mu[\alpha_1^{3*}(\alpha_1^{2*}\pi^*+(1-\alpha_1^{2*})(1+k)\pi^*)]$.

$\because\ \pi^{2*}-\pi^{3*}=g'(\overline{\alpha_1})(\alpha_1^{2*}-\alpha_1^{3*})-\mu[\alpha_1^{3*}(1-\alpha_1^{2*})k\pi^*], \alpha_1^{2*}<\overline{\alpha_1}<\alpha_1^{3*}$,

$$-\mu(1-\alpha_1^{2*})k\pi^*<g'(\overline{\alpha_1})=R(\overline{\alpha_1})-r+\overline{\alpha_1}R'(\overline{\alpha_1})+\mu\pi^*<0$$

$$-\mu(1-\alpha_1^{2*})k\pi^*(\alpha_1^{2*}-\alpha_1^{3*})>g'(\overline{\alpha_1})(\alpha_1^{2*}-\alpha_1^{3*})>0$$

$g'(\overline{\alpha_1})(\alpha_1^{2*}-\alpha_1^{3*})-\mu[\alpha_1^{3*}(1-\alpha_1^{2*})k\pi^*]<-\mu(1-\alpha_1^{2*})k\pi^*(\alpha_1^{2*}-\alpha_1^{3*})-\mu[\alpha_1^{3*}(1-\alpha_1^{2*})k\pi^*=$
$-\alpha_1^{2*}\mu(1-\alpha_1^{2*})k\pi^*<0$

So $\pi^{2*}(1)<\pi^{3*}(1)$.

Let $\pi(\beta)=\pi^{2*}(\beta)-\pi^{3*}(\beta)$, $\pi(0)=\pi^{2*}(0)-\pi^{3*}(0)>0$, $\pi(1)=\pi^{2*}(1)-\pi^{3*}(1)<0$. $\therefore\exists\beta^*$, $\pi(\beta^*)=\pi^{2*}(\beta^*)-\pi^{3*}(\beta^*)=0$. $\pi'(\beta)=\pi^{2*'}(\beta^*)-\pi^{3*'}(\beta^*)$, $\pi^{2*'}(\beta)<\pi^{3*'}(\beta)$, $\therefore\pi'(\beta)<0$, then β^* is unique, if $\beta<\beta^*$, $\pi^{2*}(0)>\pi^{3*}(0)$; if $\beta>\beta^*$, $\pi^{2*}(0)<\pi^{3*}(0)$.

7.

From the equation

$R(\alpha_1^{4*})-r+\alpha_1^{4*}R'(\alpha_1^{4*})=-\mu(\beta\pi^*+\alpha_1^{4*}(1-\beta)\pi^*+(1-\alpha_1^{4*})k\pi^*)$,

$\alpha_1^{4*}R'(\alpha_1^{4*})=-\mu(\beta\pi^*+\alpha_1^{4*}\pi^*-\alpha_1^{4*}\beta\pi^*+k\pi^*-\alpha_1^{4*}k\pi^*)-R(\alpha_1^{4*})+r$

$=-\mu(\beta\pi^*+\alpha_1^{4*}\pi^*(1-\beta-k)+k\pi^*)-R(\alpha_1^{4*})+r$

$=-\mu\alpha_1^{4*}\pi^*(1-\beta-k)-\mu\beta\pi^*-\mu k\pi^*-R(\alpha_1^{4*})+r, R'(\alpha_1^{4*})$

$=-\mu\pi^*(1-\beta-k)+\dfrac{-\mu\beta\pi^*-\mu k\pi^*-R(\alpha_1^{4*})+r}{\alpha_1^{4*}}$

$\because R(\alpha_1^{4*})-r>0$, the right side is smaller than 0, $\therefore R'(\alpha_1^{4*})+\mu\pi^*(1-\beta-k)<0$. $\alpha_1^{4*'}(\beta)=$
$\dfrac{-\mu\left(1-\alpha_1^{4*}(\beta)\right)\pi^*}{R'\left(\alpha_1^{4*}(\beta)\right)+R'\left(\alpha_1^{4*}(\beta)\right)+\alpha_1^{4*}(\beta)R''\left(\alpha_1^{4*}(\beta)\right)+\mu(1-\beta-k)\pi^*}$

$\therefore\alpha_1^{4*'}(\beta)>0$.

8.

In the case of parallel investing

$$R(\alpha_1^{2*})-r+\alpha_1^{2*}R'(\alpha_1^{2*})=-\mu\beta\pi^*$$

$$\pi^{2*}=\alpha_1^{2*}(R(\alpha_1^{2*})-r)+\mu(\alpha_1^{2*}\pi^*+(1-\alpha_1^{2*})(1-\beta)\pi^*)$$

In the case of diversified investment

$$R(\alpha_1^{4*})-r+\alpha_1^{4*}R'(\alpha_1^{4*})=-\mu(\beta\pi^*+\alpha_1^{4*}(1-\beta)\pi^*+(1-\alpha_1^{4*})k\pi^*)$$

The profits of one bank are
$\pi^{4*} = \alpha_1^{4*}(R(\alpha_1^{4*}) - r) + \mu[\alpha_1^{4*}(\alpha_1^{4*}\pi^* + (1-\alpha_1^{4*})(1+k)\pi^*) + (1-\alpha_1^{4*})(\alpha_1^{4*} \times 0 \times \pi^* + (1-\alpha_1^{4*})(1-$
$\beta\pi*$
$= \alpha_1^{4*}(R(\alpha_1^{4*}) - r) + \mu[\alpha_1^{4*}(\pi^* + (1-\alpha_1^{4*})k\pi^*) + (1-\alpha_1^{4*})(1-\beta)\pi^* - \alpha_1^{4*}(1-\alpha_1^{4*})(1-\beta)\pi^*]$
$= \alpha_1^{4*}(R(\alpha_1^{4*}) - r) + \mu[\alpha_1^{4*}\pi^* + (1-\alpha_1^{4*})(1-\beta)\pi^* - \alpha_1^{4*}(1-\alpha_1^{4*})(1-\beta-k)\pi^*]$
Let $g(\alpha) = \alpha(R(\alpha) - r) + \mu(\alpha\pi^* + (1-\alpha)(1-\beta)\pi^*)$

$$g'(\alpha) = R(\alpha) - r + \alpha R'(\alpha) + \mu\beta\pi^*$$

$g''(\alpha) = R'(\alpha) + R'(\alpha) + \alpha R''(\alpha) < 0$. Then $\pi^{2*} - \pi^{4*} = g(\alpha_1^{2*}) - g(\alpha_1^{4*}) + \mu\alpha_1^{4*}(1-\alpha_1^{4*})(1-\beta-$
$k\pi*$,
$g(\alpha_1^{2*}) - g(\alpha_1^{4*}) = g'(\overline{\alpha_1})(\alpha_1^{2*} - \alpha_1^{4*})$, $\alpha_1^{2*} < \overline{\alpha_1} < \alpha_1^{4*}$.
$\because g'(\overline{\alpha_1}) = R(\overline{\alpha_1}) - r + \overline{\alpha_1}R'(\overline{\alpha_1}) + \mu\beta\pi^*$, $g'(\alpha_1^{2*}) = 0$, and $g''(\alpha) < 0$, $\therefore g'(\overline{\alpha_1}) < 0$. Then $g(\alpha_1^{2*}) - g(\alpha_1^{4*}) = g'(\overline{\alpha_1})(\alpha_1^{2*} - \alpha_1^{4*}) > 0$. If $\beta < 1-k$, $\mu\alpha_1^{4*}(1-\alpha_1^{4*})(1-\beta-k)\pi^* > 0$, $\therefore \pi^{2*} - \pi^{4*} > 0$.

9.

$$\Pi(\beta) = 2\alpha_1^{2*}(\beta)\left(R\left(\alpha_1^{2*}(\beta)\right) - r\right) + 2\mu\pi^*$$

$$\Pi'(\beta) = 2\left(R\left(\alpha_1^{2*}(\beta)\right) - r + \alpha_1^{2*}(\beta)R'\left(\alpha_1^{2*}(\beta)\right)\right)\alpha_1^{2*'}(\beta)$$

$R\left(\alpha_1^{2*}(\beta)\right) - r + \alpha_1^{2*}(\beta)R'\left(\alpha_1^{2*}(\beta)\right) = -\mu\beta\pi^*$, $\therefore \Pi'(\beta) < 0$. So $\Pi(\beta)$ is decreasing in β.

10.

In a similar way, $E_{R\alpha}\left(\alpha_1^{4*}(\beta)\right)$ is increasing in β. Suppose that numerically $\exists\bar{\bar{\beta}}$, $E_{R\alpha}\left(\alpha_1^{4*}(\bar{\bar{\beta}})\right) = 1$, and $W'(\bar{\bar{\beta}}) = 0$, then $\bar{\bar{\beta}}$ is unique. If $\beta < \bar{\bar{\beta}}$, $W'(\beta) > 0$, the social welfare is decreasing in β; if $\beta > \bar{\bar{\beta}}$, $W'(\beta) < 0$, the social welfare is decreasing in β.

$\because \alpha_1^{4*}(\beta) > \alpha_1^{2*}(\beta)$, $E_{R\alpha}\left(\alpha_1^{2*}(\bar{\beta})\right) = 1$, then $E_{R\alpha}\left(\alpha_1^{4*}(\bar{\beta})\right) > 1$. $\because E_{R\alpha}\left(\alpha_1^{4*}(\bar{\bar{\beta}})\right) = 1$, and $E_{R\alpha}\left(\alpha_1^{4*}(\beta)\right)$ is increasing in β, $\bar{\bar{\beta}} < \bar{\beta}$.

11.

$$W(\bar{\beta}) = 2\alpha_1^{2*}(\bar{\beta})\left(R\left(\alpha_1^{2*}(\bar{\beta})\right) - r\right) + 2\mu\pi^* - 2\left(1 - \alpha_1^{2*}(\bar{\beta})\right)r$$

$$W(\beta^* + \Delta\beta) < 2\alpha_1^{4*}(\beta^*)\left(R\left(\alpha_1^{4*}(\beta^*)\right) - r\right) + 2\mu\pi^* - 2\left(1 - \alpha_1^{4*}(\beta^*)\right)r$$

$W(\bar{\beta}) - W(\beta^* + \Delta\beta) > 2\left(\alpha_1^{2*}(\bar{\beta})R\left(\alpha_1^{2*}(\bar{\beta})\right) - \alpha_1^{4*}(\beta^*)R\left(\alpha_1^{4*}(\beta^*)\right)\right) = 2\left(R(\bar{\alpha}) + \bar{\alpha}R'(\bar{\alpha})\right)\left(\alpha_1^{2*}(\bar{\beta}) -\right.$
$\alpha 14*\beta*=2R\alpha 1-ER\alpha\alpha\alpha 12*\beta-\alpha 14*\beta*$

$\alpha_1^{2*}(\bar{\beta}) < \bar{\alpha} < \alpha_1^{4*}(\beta^*)$. $\because \bar{\bar{\beta}} < \bar{\beta} \leq \beta^*$, $\therefore 1 - E_{R\alpha}(\bar{\alpha}) < 0$, $W(\bar{\beta}) - W(\beta^* + \Delta\beta) > 0$.

12.

$W(\beta^*) = 2\alpha_1^{2*}(\beta^*)\left(R\left(\alpha_1^{2*}(\beta^*)\right) - r\right) + 2\mu\pi^* - 2\left(1 - \alpha_1^{2*}(\beta^*)\right)r$, $W(\beta^* + \Delta\beta) \approx 2\alpha_1^{4*}(\beta^*)\left(R\left(\alpha_1^{4*}(\beta^*)\right) - r\right) + 2\mu\pi^* - 2\left(1 - \alpha_1^{4*}(\beta^*)\right)r$, $W(\beta^*) - W(\beta^* + \Delta\beta) \approx 2R(\bar{\alpha})\left(1 - E_{R\alpha}(\bar{\alpha})\right)\left(\alpha_1^{2*}(\beta^*) - \alpha_1^{4*}(\beta^*)\right)$, $\alpha_1^{2*}(\beta^*) < \bar{\alpha} < \alpha_1^{4*}(\beta^*)$. $\because \bar{\bar{\beta}} \leq \beta^* < \bar{\beta}$, so we do not know whether $1 - E_{R\alpha}(\bar{\alpha})$ is larger, smaller or equal to 0, then $W(\beta^*)$ can not compare with $W(\beta^* + \Delta\beta)$, the relationship depends on the value of $\bar{\bar{\beta}}$ and $\bar{\beta}$.

13.

$$W(\bar{\bar{\beta}}) = 2\alpha_1^{4*}(\beta^*)\left(R\left(\alpha_1^{4*}(\beta^*)\right) - r\right) + 2\mu\pi^* - 2\left(1 - \alpha_1^{4*}(\beta^*)\right)r$$

$$W(\beta^*) - W(\bar{\bar{\beta}}) = 2R(\bar{\alpha})\left(1 - E_{R\alpha}(\bar{\alpha})\right)\left(\alpha_1^{2*}(\beta^*) - \alpha_1^{4*}(\bar{\bar{\beta}})\right)$$

$\alpha_1^{2*}(\beta^*) < \bar{\alpha} < \alpha_1^{4*}(\bar{\bar{\beta}})$, $\because \beta^* < \bar{\bar{\beta}} < \bar{\beta}$, $\therefore 1 - E_{R\alpha}(\bar{\alpha}) > 0$, then $W(\beta^*) < W(\bar{\bar{\beta}})$.

14.

If the bank purchases the credit protection, the expected profits in the second period are $\alpha_1^{5*}R\left(\alpha_1^{5*}\right) - r$. If not, the expected profits are $\pi^* = \alpha^*\left(R(\alpha^*) - r\right)$, $\alpha_1^{5*} > \alpha^*$. $\alpha_1^{5*}R\left(\alpha_1^{5*}\right) - r < \alpha_1^{5*}R\left(\alpha_1^{5*}\right) - \alpha_1^{5*}r = \alpha_1^{5*}\left(R\left(\alpha_1^{5*}\right) - r\right) \leq \alpha^*\left(R(\alpha^*) - r\right)$. The profits if not purchase the protection are more, so the bank does not purchase the protection in the second period.

15.

The first order condition in the case of purchasing credit protection is

$$R\left(\alpha_1^{5*}\right) + \alpha_1^{5*}R'\left(\alpha_1^{5*}\right) = 0$$

The profits are

$$\pi^{5*} = \alpha_1^{5*} R\left(\alpha_1^{5*}\right) - r + \mu\pi^*$$

The first order condition in the case of parallel investing is

$$R(\alpha_1^{2*}) + \alpha_1^{2*} R^{'}\ (\alpha_1^{2*}) = r - \mu\beta\pi^*$$

The profits are

$$\pi^{2*} = \alpha_1^{2*}(R(\alpha_1^{2*}) - r) + \mu(\alpha_1^{2*}\pi^* + (1-\alpha_1^{2*})(1-\beta)\pi^*)$$

When we compare the parallel investing and purchasing protection in the cases of $\beta = 0$ and $\beta = 1$.

1) If $\beta = 0$, $R(\alpha_1^{2*}) + \alpha_1^{2*} R^{'}\ (\alpha_1^{2*}) = r$, that is $\alpha_1^{2*} = \alpha^*$.

$$\pi^{2*} = \alpha^*(R(\alpha^*) - r) + \mu\pi^* = (1+\mu)\pi^*$$

$$\pi^{5*} = \alpha_1^{5*} R\left(\alpha_1^{5*}\right) - r + \mu\pi^* \leq \alpha_1^{5*}\left(R\left(\alpha_1^{5*}\right) - r\right) + \mu\pi^*$$

$$\pi^{2*} - \pi^{5*} \geq \alpha^*(R(\alpha^*) - r) - \alpha_1^{5*}\left(R\left(\alpha_1^{5*}\right) - r\right) > 0$$

2) If $\beta = 1$, $R(\alpha_1^{2*}) + \alpha_1^{2*} R^{'}\ (\alpha_1^{2*}) = r - \mu\pi^*$,

$$\pi^{2*} = \alpha_1^{2*}(R(\alpha_1^{2*}) - r) + \mu\alpha_1^{2*}\pi^*$$

$$\pi^{2*} - \pi^{5*} = \alpha_1^{2*}\ R(\alpha_1^{2*}) - \alpha_1^{2*}\ r + \mu\alpha_1^{2*}\pi^* - \alpha_1^{5*} R\left(\alpha_1^{5*}\right) + r - \mu\pi^*$$

$$= \alpha_1^{2*}\ R(\alpha_1^{2*}) - \alpha_1^{5*} R\left(\alpha_1^{5*}\right) + (1-\alpha_1^{2*})r - (1-\alpha_1^{2*})\mu\pi^*$$

$$= \left(R(\bar{\alpha}) + \bar{\alpha}R^{'}\ (\bar{\alpha})\right)\left(\alpha_1^{2*} - \alpha_1^{5*}\right) + (1-\alpha_1^{2*})(r - \mu\pi^*)$$

$\bar{\alpha} \in \left(min\left(\alpha_1^{2*}, \alpha_1^{5*}\right), max\left(\alpha_1^{2*}, \alpha_1^{5*}\right)\right)$.

i. If $r - \mu\pi^* < 0$, $\alpha_1^{2*} > \alpha_1^{5*}$, $R(\bar{\alpha}) + \bar{\alpha}R^{'}\ (\bar{\alpha}) < 0$, $\pi^{2*} - \pi^{5*} < 0$.

ii. If $r - \mu\pi^* \geq 0$, $\alpha_1^{2*} \leq \alpha_1^{5*}$, $r - \mu\pi^* \geq R(\bar{\alpha}) + \bar{\alpha}R^{'}\ (\bar{\alpha}) \geq 0$, and $0 < \left|\alpha_1^{2*} - \alpha_1^{5*}\right| < 1 - \alpha_1^{2*}$, so $\pi^{2*} - \pi^{5*} \geq 0$.

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