SCHOOL AND CULTURAL INFLUENCES ON MATHEMATICS ACHIEVEMENT

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Abstract

Students' mathematics achievement is influenced by many factors. As such, the performance gap in mathematics between students from different countries was attributed in turn, to the teachers' subject matter and pedagogical knowledge, curriculum development, native language, as well as parental teaching strategies. While research studies provided reasonable explanations for the performance gap, their limitation was to only analyze isolate factors believed to influence mathematics learning. This review challenges findings of current research studies, which analyzed these factors in isolation, advocating for developing a new lens to look at comparative studies, a more complex approach that considers the interactions of the many factors believed responsible for the mathematics success of a particular group of students.

Key words: Achievement gap, comparative education, cross-cultural studies, mathematics education

Introduction

It is an established fact that Chinese students tend to outperform other countries in the international mathematics competitions, while the American students' performance has been described as merely average: "when twelfth-grade American students were compared to students from fourteen other countries, they were the in lowest quarter in geometry, and in algebra they were second from the bottom" (Stevenson & Stigler, 1992, p.31). Results of the Third International Mathematics and Science Study (TIMSS), conducted in 1994 and repeated in 1999, 2003, and 2007 still showed that Chinese students outperformed the United States students at all levels of assessment in mathematics: fourth-, eighth-, and twelfth-grades (National Center for Educational Statistics, 1994; 1999a; 1999b; TIMSS 2003 International Mathematics Report; TIMSS 2007 International Press Release; Programme for International Student Assessment (PISA) 2000). Other cross-cultural comparisons of mathematics conducted on a lower scale also revealed the better performances of Chinese students at different levels (Brenner, Herman, Ho, & Zimmer, 1999; Cai, 2000; Huntsinger, Jose, Larson, Krieg & Shaligram, 2000; Stevenson, & Stigler, 1992), attempting to justify this better performance in the light of a series of factors that may impact student mathematics learning.

The general assumption regarding student mathematics performance is that it depends on a series of factors, such as: teacher content knowledge, instructional practices, parental teaching strategies, curriculum, and language, to only name a few factors (Brenner et al., 1999; Cai, 2000; Huntsinger et al., 2000; Ma, 1999; Miura, 1987; Stevenson & Stigler, 1992). As such, the success or lack of success of students in the international mathematics competitions came to be regarded as a direct result of these aforementioned factors. Comparative literature reviews provided thorough analyses of factors believed to be responsible for the performance gap among students from different countries. Some researchers and large-scale studies showed that the school factors widened the gap in performance (National Center for Educational Statistics, 1994, 1999a; 1999b; Perry, 2000; PISA 2000).

Firstly, teacher content knowledge is believed to influence the way teachers transfer this knowledge into classrooms (Ball, 1990; Ball & McDiarmid, 1990; Kennedy, 1991; Schulman, 1986) and further impact student learning and achievement. Hence, the stronger mathematical content knowledge and pedagogical content knowledge of the Chinese elementary school teachers in Ma's (1999) study may lead to a better understanding of basic mathematics of Chinese students and in turn, it may lead to their better performance in the international mathematics competitions. This advantage, coupled with a more cohesive Chinese curriculum (Li, 2000), which exposes Chinese students to fewer but more in-depth topics than the American textbooks, may lead to the better performance of Chinese students in the international mathematics competitions.

Secondly, researchers (Miura, 1987) advocated that differences in mathematics performance are innate in the language we speak: speakers of Asian languages seem to have an advantage over speakers of other languages (English, French) due to their numerical number characteristics congruent with the Base-10 system.

The Base-10 system impacts the acquisition of place value concepts, which in turn impacts all other mathematics concepts, like addition, subtraction, multiplication (Ho & Cheng, 1997).

Thirdly, other researchers (Chen & Stevenson, 1995; Huntsinger et al., 2000) stated these differences are perpetuated by the cultural milieus in which students live, such as more liberal/authoritative households (Caucasian American), versus the more traditional/authoritarian households (Asian American). As such, students in more traditional households were exposed to more formal types of interactions (drills, worksheets and rubrics) that seemed to benefit the Asian American students in Huntsinger et al.'s study.

One limitation the above studies share is the analysis of the factors believed to impact student mathematics performance in isolation, which may only provide a limited understanding of the Chinese students' success. A closer look at the TIMSS (1994; 1999) scores revealed that if Hong Kong students were compared to students from Romania, although the latter shared some of the characteristics believed to render the Chinese students successful, the Romanian students performed significantly lower than the Chinese students and the U.S. students. For example, Romanian curriculum was as cohesive as the Chinese curriculum (Schmidt et al., 2001), while the overall school climate in Romania was again similar to the Chinese school climate (TIMSS, 1994; 1999). Introducing a different variable in the equation might therefore reveal interesting findings, which may challenge the results of current research studies comparing the U.S. only to the top performing countries.

In an attempt to fill the gap in the comparative literature (Wang & Lin, 2005), namely that existing studies fail to provide a deep understanding of the factors influencing student mathematics achievement, this review of literature proposes to show the limitations of only looking at the factors that influence student mathematics achievement in isolation, reiterating the need to look at the interrelatedness of the multiple factors. Moreover, this review strives to challenge previous research findings that held particular factors responsible for the mathematics achievement of Asian and United States students, by introducing in the equation a third variable: the Romanian educational system. Due to its similarities and differences to both China and United States, Romania has the potential to challenge the previous assumptions that either home or schooling factors are responsible for the better performance of Asian students.

Conceptual Framework

The theoretical framework that guided this literature review and prompted the author to account for different types of interactions between factors considered responsible for student learning is the complexity theory. A science of emergence (Waldrop, 1992), complexity is a class of behaviors in which the components of a system constantly organize themselves into larger structures: "each agent finds itself in an environment produced by its interactions with the other agents in the system, and because of the constant reaction to the other agents' action, nothing in the environment of the complex systems is fixed" (Waldrop, 1992, p. 145). The interactions between these independent agents are "subject to ongoing co-adaptations" (Davis & Sumara, 2006, p. 11). These complex systems have also been named autopoetic systems (Maturana & Varela, 1984), which function as a composite unity, and the relations between these components constitute the organization of the system. Senge (1990) defined this concept as "systems thinking," advocating that everything that happens in a living system is caused by the actions of all the factors involved in that particular action. The major assumption in complexity theory is acknowledging that there are no independent agents, that each agent is part of a team, and that disregarding this fact may only provide limited understanding of how systems evolve.

Researchers (Davis & Simmt, 2006; Davis & Sumara, 2006) discussed the implications applying the complexity lens in the field of mathematics education: in teaching mathematics, teachers need to consider the interactions between multiple agents, like individual understanding, collective knowledge, curriculum structures, classroom collectivity (Davis & Sumara, 2006).

Moreover, Davis & Simmt (2006) regard complexity as a source of advice for mathematics teachers, who should be no longer thinking in terms of "What's happening?," but in terms of "How can it be made to happen?" impacting both teaching and learning of mathematics:

> The manner in which complexity science casts both children and mathematics-along with classroom communities, conversational interactions, and similar phenomena-as adaptive and self-organizing learning forms prompts us to think in terms of unities nested within unities, rather than in terms of discrete entities.

Mathematics teaching and learning needs therefore to be considered from a complex perspective, in order to have a more accurate understanding of the factors responsible for the achievement gap, as only analyzing a country's curriculum, teaching strategies or home influences in isolation may not paint the complete picture of why Asian students outperform other nations in mathematics. From the perspective of complex theory, the researcher investigated both schooling factors (curriculum and teaching practices) and non-schooling factors (language and home influences) and their impact on student mathematics knowledge in this review of relevant literature. The researcher understands the complexity of such a topic, and the fact that many other schooling and non-schooling factors may be responsible for mathematics achievement, besides the ones analyzed in this review. However, to be able to address all the cultural and school factors in a limited space is beyond the scope of a review paper. The choice of the two main school factors analyzed lies in the significance they each played in the mathematics education reform in the U.S., at both a curricular and at an instructional level. Starting from the late 1990s advocates of reform blamed the poor results of the United States students in mathematics on the traditional teaching practices and fragmented mathematics curriculum and suggested reform should start particularly in these areas (Nicol, 1999; Spungin, 1996; Stedman, 1997a; Stedman, 1997b).

Almost a decade later, Ball (2003), Lawrenz (2003), and Manoucheri & Goodmman (2000) still advocated reform in an effort to improve the teaching and learning of mathematics. These reform proposals seemed quite logical. Since the gap in the mathematics performance between United States students and students from the top-performing countries was mainly explained through large discrepancies in national curriculum and teaching practices, it was deemed important that the reform of the United States educational system start in these two areas (Ball, 2003; Senger, 1999). The choice of the non-schooling factors, namely the linguistic and home influences (parental attitudes and strategies to help students learn mathematics at home), besides the obvious positive impact parental involvement has on student achievement (Gonzalez, 2002; Presinni, 1998) is also one of convenience, due to the fact that comparative studies of language and parental expectations and strategies included the student population of the three studies this research is focusing on: China, Romania and the U.S. Chinese and U.S. students were at the center of these comparisons, as the Chinese students consistently performed well in the international mathematics competitions, and the factors that seemed to influence the better performance of these students were analyzed in an attempt to understand the performance gap.

A third group of students was introduced in this review, the Romanian students. This group of students presents similarities and differences to both the U.S. and the Chinese students, and a look at their performance may shed more light into the factors believed to influence the better performance of the Chinese students. Moreover, introducing the Romanian students as a new variable may challenge the current research findings of studies that consistently compared the results of the U.S. students to those of the Chinese students, according to which the success of the Chinese students may be explained by a centralized curriculum, the linguistic advantage, better parental expectations, etc. While analyzed in isolation, these factors may explain the better results of the Chinese students, but they fail to do the same for the lack of success of the Romanian students in the international mathematics competitions (Author, 2008; Wang, J., Author, Sas, M., & Lin, E. , 2008; Wang & Lin, 2005).

Methodology

There were 34 studies discussing the impact of schooling and non schooling factors on student mathematics achievement meeting the criteria for inclusion in this review, of both quantitative and qualitative nature. The author used the following electronic databases to conduct the review: (a) the Eric database, (b) the Education database, and (c) the Ebsco database for the years 1980-2007. These databases were searched using the following terms: (a) cross-cultural comparisons of mathematics; (b) language and mathematics achievement; (c) cultural influences on mathematics achievement; (d) instructional practice and mathematics. In addition, the author used two meta-analyses, Ball & McDiarmid (1990) and Kennedy (1991) and included the following books in the review: Ma (1999); Maturnana & Varela (1984) Stevenson & Stigler (1992); Schmidt et al., (2001); Senge (1990); Valverde, Bianchi, Wolfe, Schmidt & Houang (2002); Waldrop (1992). One unpublished dissertation (Author, 2008) was also included in the review as it discusses findings about Romanian students. Due to its potential to conduct cross- cultural comparisons, the TIMSS study database was analyzed for the years 1994-2007, and the PISA study database for the year 2000.

The following criteria were used in the selection of the studies for this review: firstly, due to the scope of this review, the largest part of the studies comprised in this review included cross-cultural comparisons of students from China, Romania and the U.S. Secondly, in order to avoid generalization to the larger population of Asian students, this review only included those studies that reported the results of Chinese students, including Korean and Japanese students only when Chinese students were compared with them. Thirdly, in using the TIMSS and PISA databases, and in consistency with the other studies used in this review, the author discussed the results of the following three countries: China (Hong Kong), Romania and the U.S. The selection of the Hong Kong students emerged as a necessity due to the fact that students from Mainland China did not participate in any of the TIMSS studies, and also because Chinese Taipei students were not part of the TIMSS 1994 study. Moreover, the analysis was conducted only at the eight-grade level, due to the fact that Romanian students only participated at the eight-grade level in all the TIMSS mathematics studies.

Fourthly, isolated studies were also included in this review only if they could share more light regarding the educational system of a country that has not been the scope of much research (Romania), in order to better explain the performance of the Romanian students in the international mathematics competitions.

The researcher analyzed the findings of these studies, as well as compared the results of students in the TIMSS and PISA studies, discussed their assumptions and challenged their findings in light of the complexity theory. If particular factors, deemed responsible for the success of the Chinese students in the international mathematics competitions, cannot explain the lack of success of Romanian students (who share a lot if similar characteristics in terms of factors that would place the Chinese students at an advantage in mathematics learning), how reliable are the results of studies that analyzed these factors in isolation?

Findings

Cultural Influences on Student Achievement

Language and mathematics achievement: a theory of linguistic relativity.

Over the years, researchers advocated pro and against the advantages of a national language for student learning of mathematics. One of the groundbreaking researchers in this field, Miura (1987) argued in favor of the linguistic influence on the mathematics achievement of young students. In an attempt to look at how children from different cultures speaking different languages construct numbers, Miura, Kim, Chang & Okamoto (1988) tested American, Chinese, Japanese and Korean first graders and Korean kindergartners, asking them to construct five numbers using Base-10 blocks. The results revealed a greater mental flexibility for all the Asian students who were able to construct numbers in two ways on a larger scale than did the American students.

How does language increase/decrease the understanding of mathematical concepts? According to Miura et al., (1988) “through language, numbers are mentally represented and stored, and for those languages that are rooted in ancient Chinese (Chinese, Japanese and Korean), numerical names are organized so that they are congruent with the traditional Base-10 numeration system” (Miura, p.1446). In these languages, the value of a given digit or multi digit numeral depends on “the face value of the digit (0 through 9) and on its position on the numeral, with the value of its position increasing by powers of 10 from right to left” (Miura, p.1446). Conversely, English speakers learn the words to 20 by rote, many of the number words having no initial meaning (Fuson, 1991). In an attempt to further investigate whether or not other non-Asian speakers would perform similarly to United States students, Miura et al., (1994) replicated the same study including students from France and Sweden along with Asian first grade students (Chinese, Japanese and Korean) and American first grade students. Results again showed a preference of Asian speakers for canonical and noncanonical constructions of tens and ones rather than one-to-one unit constructions. Differences were also found in the ability of the Asian students to construct two correct constructions for the same number, which may reveal a greater flexibility with number quantity.

Acknowledging the differences in cognitive representations of numbers between speakers of Asian and non-Asian languages, other researchers further investigated the impact of the cognitive representations of number on specific mathematical concepts such as counting. Miller, Smith, Zhu & Zhang (1995) tested their assumption that differences in counting ability between Chinese and American pre-school students should focus on areas in which languages differ, namely the “teen” names.

Results confirmed the hypothesis that United States students had trouble learning the numbers between 10-20, which may be an indicator of their failure to understand the Base-10 system, and consequently, they may start school with a disadvantage that arises from the way English counting system is constructed. While investigating the practices used in American schools to remedy the disadvantage represented by the lack of verbal support in the English language for multi-unit Base-10 representations, Fuson & Kwon (1991) found out that children are taught multi-digit addition and subtraction as step-by-step procedures of adding and subtracting single-digit numbers, and as such, students view multi-digit numbers as composed of single-digits placed to each other. School does little to provide students with the support needed in order to fill this gap believed to be the direct result of language. One solution proposed by Miller et al., (1995) in order to remedy this disadvantage is to familiarize American children with Arabic numerals at an earlier age, as these numerals provide a consistent Base-10 representation for numbers. While all these studies provided another explanation of why Chinese students performed at a superior level in the mathematics competitions, their limitation was the exclusion of a series of different factors, which, along with language, may justify the better mathematics results of the Asian students in the international comparisons in later grades (i.e. kindergarten exposure, family influences).

Understanding the influence of language on mathematics achievement and the fact that non-Asian speakers may begin school with a disadvantage may impact the way curriculum is designed and teaching is approached. In an effort to probe for linguistic relativity and to investigate whether or not language alone is responsible for the gap in performance between different groups of students, Wang, Lin, Author, Sas (2008) replicated Miura's (1987) study in terms of methodology, testing Chinese, Romanian and American students. Romanian language has been identified as an important variable to test linguistic influences on students' mathematics achievement due to its similarity to both Chinese and French, languages previously tested in Miura et al.'s (1994) study. Romanian matches the ancient Chinese-based languages in its numerical language characteristics, but not in its linguistic roots. Its roots are Latin, like French. Romanian language is, therefore, unique in its similarity and difference to the Chinese-based languages in that it allows to some extent for an isolation of the numerical language characteristics variable.

Results showed no statistical significant differences between Romanian students' performance and that of United States students, which is a surprising factor since English number naming is not consistent with base-10 system. On the other hand, Chinese children outperformed both Romanian and United States students in using base-10 systems. Moreover, the Chinese students' performance increased substantially as they progressed from the first to the second trial. This study showed significant differences in the number manipulation of students from three different countries, implying a connection between language and other factors that may impact students' mathematics achievement. As such, despite the semi-consistency and transparency with the base 10 numbers of the Romanian language, the Romanian students performed closer to their American peers than to the Chinese peers, as initially believed. If the language advantage did not seem to be a factor in the way Romanian students manipulated numbers, then what other factors may be involved in the Chinese students' better manipulation of numbers? Can it be that the better subject matter knowledge of the Chinese teachers as well as a possible better home support in the Chinese families may reinforce the language advantage?

Home influences on students' mathematics achievement.

Despite the significance of the impact language may have on student mathematics learning, the above studies showed that the linguistic factor alone cannot fully explain the achievement gap in mathematics in the China-Romania-US comparison. Other cultural factors, along with the linguistic influence, are therefore believed to impact student mathematics achievement. How parents raise their children and teach them mathematics may determine how proficient their children are in mathematics, as the below studies show.

Parental Strategies and Mathematics Achievement

In an attempt to fill the gap in the literature regarding what teaching practices are most common at home among parents with diverse cultural backgrounds, Huntsinger et al. (2000) investigated parental practices in 40 Chinese-American and 40 Caucasian-American homes and correlated these practices to the better mathematics performance of Chinese-American students. Findings revealed that Chinese American students outperformed their Caucasian peers in mathematics, and this better performance of Chinese-American students might be due to the more formal approaches used by their parents.

Formal methods of teaching were reported to be: longer duration of interactions between parents and students, paying more attention to the written representation of a problem, expecting children to spend greater amounts of time in studying mathematics, using memorization, drills, and worksheets. Not only Chinese-American students who were taught by their parents with more formal strategies performed higher, but also the Caucasian students who were taught with the same approaches scored higher than those who were taught using more informal techniques. A look at the way Romanian parents interact with their children at home reinforces previous research findings: more formal interactions between parents and children are more likely to lead to a better understanding of the base 10 concepts. Results of a study (Author, 2008) investigating sixty-four first-grade Romanian students' understanding of place value concepts revealed that if parents helped their children understand these concepts both before and after the teacher taught them, children's understanding of place value was stronger. In order to help their children understand place value concepts before class, parents played games that involved counting (Monopoly), practiced all the time and gave children a lot of examples with numbers. After the teacher addressed the concepts in class, parents stated they checked their children's homework to have an understanding of what happened in class, created their own worksheets and used different resources to consolidate the concepts taught in class.

This more formal interaction (Huntsinger et al., 2000), in which parents spent a longer amount of time with their children, developing their own worksheets and drilling their children more, resulted in a better student understanding of base 10 numbers in the current study, which is evidenced in the final test scores, as 65 % of the students in one class scored A even when tested on more complex notions. On the other hand, the parents in the other three classrooms mostly followed the teachers' and the textbook's model and were less confident of the level of support provided, as they stated they did "as much as they could" or "not a good job" in explaining the concepts. Consequently, the parents who got involved more at home in their children's learning, going beyond the teacher's explanations and model, tended to provide their children with the support needed to understand base 10 numbers.

Parental Expectations, Student Motivation and Mathematics Achievement

Chen & Stevenson (1995) compared and contrasted beliefs and attitudes about education of 1,958 American students, 2,600 East Asian students from China and Japan, and 304 Asian American students. Data were drawn from student questionnaires and a test with open-ended questions. Overall findings revealed that Asian-American students outperformed their Caucasian peers in all the trials, but they performed lower than both the Chinese and the Japanese students. This study suggests that parental expectations and student standards, along with student motivation impact mathematics achievement. As such, students who set higher standards for themselves, like going to college (East Asian students) scored generally higher than students who were motivated to get a better job (Asian-American students). On the other hand, other achievement-related behaviors (i.e. time allocated to studying mathematics at home) could not provide a clear explanation for the better results in mathematics achievement of different groups, since the students who scored the highest (both Chinese and Japanese) dedicated less time to mathematics than did Asian American students.

This study supports the cultural-motivational theory of academic achievement, namely that the beliefs and attitudes of students lead to high motivation and that high parental standards represent a cultural heritage characteristic in Asian students in general. However, immigration and acculturation to new settings, as in the case of Asian-American students, may produce differences in this cultural heritage, distinguishing these students from both their Asian and Caucasian peers. This study only analyzed students' attitudes about mathematics, and it hints at what parents expect of their children without looking more in-depth at what parental practices may account for gaps in performances across cultures and among groups with the same cultural background.

In the same vein, Hess, Chih-Mei & McDevitt (1987) argued in favor of beliefs held by parents in different cultures and their impact on students' performance, specifically the role of success in the students' mathematics achievement. Data were drawn from interviews with fifty-one Chinese-American mothers and their children, forty-seven Chinese mothers from People's Republic of China (PRC) and their children, as well as forty-eight American mothers and their children. Findings revealed significant differences in the factors perceived to impact mathematics achievement. The PRC families tended to attribute failure to causes under their control; the other two groups believed mathematics failure was due to factors over which they had no control (i.e. ability, luck, school training). Cultural variations were also present in the groups' beliefs about the better mathematics performance.

Mothers and children from PRC gave most of the credit to schools, Chinese Americans gave credit to home, while their Caucasian peers regarded home training almost as significant as school training. An interesting conclusion arises: school, more than home training is believed to be responsible for the better results of both the Asian students and the Caucasian American families (Chen & Stevenson, 1995). On the other hand, if school (teachers and curriculum) is assumed to impact the students' success in mathematics positively even more so than home training, how may the gap in mathematics performances of students from different cultural groups be explained, if Romanian and Chinese students in the TIMSS study (1994, 1999) seem to share similarities in terms of school culture (hours of mathematics instruction, instructional practices, curriculum)? TIMSS (1994,1999) also analyzed differences in beliefs about success and failure in mathematics between students from 41 countries. Significant findings revealed that there was no correlation between parental and student expectations to perform well in mathematics and students' mathematics achievement. To exemplify, note that 98% of the United States parents expected their children to do well in mathematics (they were situated in 28th place out of 41 countries), whereas 93% of the Chinese parents expected their children to do well in mathematics and they were situated in fourth position. Interestingly enough, also 93 % of the Romanian parents expected their children to do well in mathematics, yet Romanian students ranked in the 34th place.

Are these beliefs about mathematics supported by home study? Paradoxically, Romanian students tend to spend more time at home studying mathematics than do their counterparts from China and United States. While the Chinese students spent 0.9 hours a day studying mathematics, and the U.S. students 0.8 hours per day, Romanian students dedicated an average of 1.8 hours per day to the study of mathematics, which may be correlated to their expectation to perform well in mathematics. Very interesting findings are also revealed from the investigation of students' attitudes about mathematics. Discrepancies have been found in the students' interest in mathematics and their mathematics results. Only 48% of the Chinese students liked mathematics, while 23% disliked it. Compare these findings to the attitudes of American students (47% liked mathematics and 17% disliked it). Interestingly, more Romanian students liked mathematics (52%) and fewer disliked it (18%) than their peers from the other two countries. The overall attitudes about mathematics were also in favor of Romanian students: 60% of the students had positive attitudes, and only 25% of the students had negative attitudes about mathematics. Comparatively, less Chinese students (57%) had positive attitudes and more students (31%) had negative attitudes towards mathematics.

The paradox lies in the fact that, despite the Romanian students' positive attitudes about mathematics, parental expectations and time dedicated to the study of mathematics outside of class, 33 countries out of 41 in the TIMSS 1994 study (National Center for Educational Statistics, 1994) outperformed them. Consequently, factors that may explain the Chinese students' better mathematics performance (i.e. hard work, parental expectations, student attitudes), fail to explain the poor performance of students from other countries like Romania. Future research needs to explore the relationship between the different factors assumed to impact student success in order to provide a more plausible explanation regarding the mathematics achievement of students.

School Influences on Student Achievement

Curriculum.

Schooling factors were also believed to highly impact student achievement in mathematics. Among these, the national curriculum was deemed as essential, as it dictates what teachers teach in the classroom. Researchers (Li, 2000; Valverde et al., (2002) conducted a thorough analysis of content, topic coverage, page space dedicated to each topic and types of problems that are presented in the books, and found correlations between the concepts included in the textbooks and the mathematics achievement of the students. Using TIMSS 1994 database, Valverde et al., analyzed the structure of the textbooks, as they believed this influenced classroom experiences. They called these patterns the morphology of the book and argued that this would enable them to uncover the pedagogical model advanced by the book. There were three emergent types of books: textbooks with one dominant content theme, textbooks with more than one dominant content theme, and textbooks with fragmented content coverage. Findings reveal that United States books (58%) had a fragmented content structure, based on the repetition of the same topic spread across the book, while books from China tended to have more of a progression of sequential themes (50%). Besides analyzing the overall structure of the textbooks, researchers looked into the content coverage of the textbooks. Results indicated that the eighth-grade United States textbooks covered more topics than did the books from China.

On the other hand, 39% of the topics covered in the Chinese books had no complex performance expectations, while the median percent of the complex performance expectations was 2. For the United States books, only 5% of the covered topics had no complex performance expectations, and 15% was the median percent of the complex performance expectations (Schmidt et al., 2001). Researchers also found that analyzing the number of content strands would allow them to identify how often content themes change (that is, the number of times within each textbook that a content strand ends and a new one begins). United States textbooks were again found to have the largest number of breaks (215), while books from China had only 53 breaks. Findings indicated that the more topics changed and the larger number of topics covered (i.e. like in the case of United States textbooks), the poorer the mathematics performance of students. On the other hand, textbooks with more cohesion between topics and textbooks that covered fewer topics but more in-depth, were assumed to impact positively the mathematics achievement of the students, as in the case of Chinese textbooks.

However, this hypothesis does not provide a clear explanation for the results of the Romanian students in the TIMSS 1994 study (National Center for Educational Statistics, 1994): if curriculum were, indeed, a determining factor in what happened in the classrooms in terms of instruction and student learning, how can one explain why, despite similar curricular characteristics to Chinese (Schmidt et al., 2001), Romanian students' mathematics performance is closer to that of United States students rather than that of the Asian students? Similar to the Chinese books, Romanian books have a progression of sequential themes (50%) and cover fewer topics than do United States books (32%). Most importantly, Romanian textbooks had only 20 content breaks, less content breaks encountered in the books from the United States and China, which implies more cohesion between topics even when compared to the books from China.

Consequently, a mere curricular analysis may not provide a sufficient explanation for the better performance of the Chinese students in the international mathematics comparisons. A different curriculum analysis and interpretation is necessary to provide more insights regarding the types of topics covered and their impact on the students' achievement in mathematics. With this purpose, Li (2000) took a different approach in curriculum analysis, investigating the types of problems presented in several middle school mathematics textbooks from the United States and China. Li's hypothesis was that the difference in mathematics achievement was not made by the number of pages dedicated to each topic, but the types of problems addressed in the textbooks. To support his hypothesis, Li compared five American textbooks intended for use in the United States (in various settings and with diverse populations) and their equivalents in China.

The researcher developed a three-dimensional framework for analyzing the problems: mathematical feature, contextual feature and performance requirements, and he used this system to code all the problems that did not have accompanying solutions presented. Results showed no statistical differences for the two items (mathematical and contextual features) across the curriculum of the two countries, but significant differences in the problems' performance requirements. Findings revealed that 26% of the United States books and only 16% of the Chinese books required a conceptual understanding of the solutions, and 63% of the United States books and 72% of the Chinese books required a procedural understanding of the problems presented. Due to this study's limitation (the small number of lessons selected), the researcher recommended a future larger scale investigation of textbook problems across grade levels and content topics, viewed with dual lenses: textbook content analysis and problem analysis, that would provide better opportunities to study the effect of curriculum on students' mathematics performance.

Instructional practice.

The above studies attempted to link language, home practices and curriculum with student mathematics learning. However, the language advantages and the way a curriculum is used may only benefit students if the teacher uses this information when planning for instruction and implementing the lesson, as knowing one's students (Greenes, 1995) is assumed to be key to learning. What exactly goes on in mathematics classrooms in China, Romania and the United States, how is instruction different and how may these differences explain the better performance of the Asian students in the international comparisons? A look at the TIMSS 1994 study provides more complex examples of how mathematics classes were organized and what classroom practices were predominant in 41 different countries. As background information, the tested population for the United States was 7,087 students, for Hong Kong it was 3,339 students, and from Romania is was 3,725 students. The total number of participating schools was the following: 183 schools in the United States, 163 schools in Romania, and 85 schools in Hong Kong.

Of the above three countries only the United States and Hong Kong satisfied the guidelines for sample participation rates, grade selection and sampling procedure, whereas Romania did not meet the age/grade specifications having a higher percentage of older students, which would lead one to assume they would perform better than the other countries. When comparing the average of instructional days in school year, it was noticed that the three countries spent similar number of days teaching mathematics in the school year (Hong Kong: 171; Romania: 173; and the United States: 178) as well as a similar number of yearly mathematics instruction in hours, in the case of Romania (114 on average), and Hong Kong (118 on average). On the other hand, United States teachers spent 146 hours on average teaching mathematics in a school year. Paradoxically, despite the longer exposure to mathematics, United States students only performed average in the TIMSS 1994 and 1999 mathematics tests (National Center for Educational Statistics (1994; 1999a; 1999b). Despite a longer exposure to mathematics instruction during the school year and a slightly longer school year than in Hong Kong and Romania, more schools from the United States (60 %) reported that at least 5 % of their students were absent in a typical school day than Hong Kong (2 %) and Romania (22 %).

As seen in Chen and Stevenson's (1995) study, significant differences were found regarding the school attendance of Caucasian American, Asian American, Chinese and Japanese students. Results showed that the higher the number of days students were absent from school (Caucasian Americans), the lower the number of scores and the lower the number of days students were absent from school (Japanese and Chinese students), the higher the scores in mathematics. When comparing instructional strategies in the three countries, teachers from Hong Kong and United States were again found to approach instruction differently at the 8^{th} grade. Since Romanian students only participated at the 8^{th} grade level, with only Hong Kong and U.S. testing elementary students in these two studies, students in the same grade in the three countries were analyzed for this review. For example, if students were having difficulties, 79% of the Chinese teachers but only 21% of the United States teachers agreed to give students more practice by themselves during class. On the other hand, Romanian teachers had similar beliefs to the Chinese teachers, as 80% of them would enable students to work more in class. The structure of the classroom differs between United States and Hong Kong: fewer students worked together as a class to respond to one another in Hong Kong (11%) and Romania (12%) than do in United States (22%), but more students worked individually without assistance from the teacher in Hong Kong (62%) than in the United States (50%).

Paradoxically, some of the practices encountered in the United States classrooms should place the United States students at an advantage against students from other countries: more United States students (56%) than Chinese students (19%) discussed completed homework in their mathematics lesson almost always, and teachers checked for mathematics homework and assigned homework more frequently in the United States than in Hong Kong and Romania. When investigating classroom related practices, findings again place United States students at an advantage: more eighth-grade United States students (39%) were tested in their mathematics lessons than students from Hong Kong (37%) and Romania (35%), and they worked more from worksheets and textbooks alone in their mathematics lesson. As such, instruction seemed to be more rigorous in the United States classrooms, and it should lead to better mathematics performances. On the other hand, due to the fact that video analysis of classrooms were not conducted for neither China nor Romania, the author is limited in this review of the TIMMS results in only using the official full report, and not looking into the classroom video analysis which might provide a more thorough understanding of the performance gap.

In a similar vein, the PISA (2000) study revealed similar results to the TIMSS 1994 and TIMSS-R 1999 studies (National Center for Educational Statistics, 1994; 1999a; 1999b): Chinese students outperformed most of the countries in mathematics, with the United States students showing an average performance and Romanian students being behind. Overall results show that out of the 41 performing countries, Romania was surpassed by 31 countries and only performed similar or slightly better than 10 countries. Why do Romanian students perform so poorly in the international mathematics competitions? Despite similar instructional practices and similar school organization as the Chinese, Romania does not even perform at the United States level, which has been shown to differ significantly from the Chinese context. Consequently, more research needs to be conducted to analyze the impact of different factors on student mathematics learning, to offer plausible explanations to the performance gap evident in the three countries discussed above. A study (Author, 2008) conducted in four first-grade Romanian classrooms found that teachers who possessed a more in-depth understanding of place value concepts were generally better at planning the lesson using their past experienced teaching a similar topic.

On the other hand, the more complex objectives teachers set for their class, as well as the higher standards teachers held their students, had a positive impact on student learning and understanding of place value concepts, as the students who were engaged in both higher order and lower order thinking problems had a more in-depth understanding of place value concepts. Overall findings show that if teachers drilled more and focused on transmitting ideas in a step-by-step manner, those teachers (Ms. Ali, Ms. Reiz) had mostly a procedural understanding of the topics and conveyed a similar understanding to their students. If, on the contrary teachers possessed a strong knowledge base of place value concepts and their students and constantly tried to teach these concepts in a manner in which all their individual students would benefit, their students understood not only how to count and manipulate numbers, but why learning about these numbers were important, as in the case of Ms. Ionescu's students. Teacher subject matter knowledge seems thus vital for student learning, as evidenced by the above findings of international studies and this study on Romanian students. However, the mathematics performance gap in international studies as well as the Romanian study discussed above cannot solely be explained by differences in teachers' subject matter knowledge. Other factors need also be examined in order to provide a more in-depth understanding of why some Romanian students outperform their peers in the above study, which may explain better why Chinese and United States students outperform Romanian students in international studies.

Conclusions and Implications for Future Research

The above studies showed, to a greater or lesser extent, the impact of various factors on students' mathematics achievement. Researchers investigated teachers' content knowledge (Ma, 1999), mathematics curriculum (Li, 2000; Valverde et al., 2002), the impact of native language (Fuson, 1991; Miura et al., 1994), as well as different home influences on student achievement, such as parental teaching strategies, parental standards and expectations. Overall findings of all the above studies may shed some light on student mathematics performance. However, this understanding is limited, when comparing countries sharing similar characteristics in terms of curriculum, language influences, parental practices, classroom instruction (China and Romania) but with significant differences in terms of their students' mathematics achievement results in international comparisons. This review aimed at showing the need to consider the interactions between multiple factors, as it may lead to a more in-depth understanding of the performance of the Chinese students in the mathematics competitions, while at the same time trying to justify the average performance of the U.S. students in light of the lack of a centralized curriculum, a lack of the numerical language advantage, and weaker parental standards.

All the above factors believed to account for the success of the Chinese students in the international mathematics comparisons may be questioned if analyzed in isolation. If the centralized Chinese curriculum seems to be the answer to a better performance of the Chinese students, as the textbooks had more cohesion between topics, while covering fewer topics but more in-depth than the U.S. textbooks, the same hypothesis seems not to be true regarding the Romanian students. Following the same logic in comparing the U.S. and Chinese textbooks, Romanian books have even fewer content breaks and more cohesion between topics than the Chinese and American books. Similarly, if language was the determining factor in the Chinese students' success, in comparing the performance of the Asian students with that of their American peers who lack the advantage of the numerical characteristics of language, how can the average performance of the Romanian students be explained, despite the semi-consistency with the base 10 numbers of the Romanian language?

If the cultural-motivational theory of achievement was considered, again the better performance of the Chinese students could be explained in a comparison with the American students through a more formal home training of the Chinese parents and overall higher standards and expectations held by the Chinese parents as compared to their American counterparts. If student attitudes about mathematics were considered, more Chinese students liked mathematics (48 %) than American students (47 %), however not to a relevant degree. Still, if we considered the Romanian students in this equation, more Romanian students liked mathematics (52%) and fewer disliked it (18%) than their peers from the other two countries. Despite their positive attitudes about mathematics and time invested at home in the study of mathematics, Romanian students were outperformed by 33 other nations in the TIMSS 1994 study, which shows that cultural factors alone may not be responsible for a nation's success in mathematics.

Finally, a look at the instructional factors may also show that this factor alone may not be an accurate indicator of success in mathematics.

Romanian and Chinese teachers adopted similar instructional approaches at the 8th grade, allowing students to practice by themselves more in class (80 % and respectively 79 %), as opposed to their American counterparts (only 21 %). And even if instruction seemed to be more rigorous in the U.S. classrooms, as students discussed completed homework in their mathematics lesson almost always, and teachers checked for mathematics homework and assigned homework more frequently in the United States than in Hong Kong and Romania. Why do then the Chinese students outperform their peers in the international mathematics competitions? Is it a centralized curriculum? That may be the case, as it may be more cohesive, covering fewer topics but more in-depth. Is it the numerical language advantage? It may be as well, as the numerical language characteristics that seemed to advantage the speakers of Asian languages have a direct impact on place value concept acquisition, which is necessary for further mathematics learning. Are there parental standards and raising methods? Another viable alternative, as home influences have been shown to play a very significant role in the way children learn mathematics. Is it an altogether different and more strict way of teaching mathematics? The answer to this question may be yes again, as teacher knowledge and pedagogy have a strong influence on what and how students learn.

However, if we were to answer these questions with the performance of the Romanian students in mind, the answer to the above questions may be very different. So if it is not only the national curriculum, or the cultural influences, or pedagogy, could a more thorough understanding be provided if we considered the way these factors interacted? Thus, an analysis of how all these factors play out in the Chinese context may leave us better understanding the Chinese students performance in the international competitions than did previous studies looking at only one factor at a time.

But to better understand how these factors play a role together, future studies need to focus on analyzing the interaction of these various factors on students' mathematics achievement. Accordingly, the need for a complexity theory of interrelated factors appears to be important to providing a more in-depth understanding of student mathematics achievement. Looking at the way multiple factors interact and influence student learning may suggest why some students perform better than other students even when exposed to a similar curriculum and teaching practice. Moreover, the possible similarities and differences related to the case of Romania in curriculum and instructional practices, as shown by the TIMSS data, may lead to a call for further investigations regarding mathematics teaching and learning in Romania.

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