Clustering of Decimal Misconceptions in Primary and Secondary Classes

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Abstract

The purpose of this study was to build on existing research about primary and secondary students' thinking about decimals. One of the investigations in this study was on the clustering of decimal misconception codes in classes and the aim was to establish variations by class across the primary (Grade 6 only) and secondary (Grades 7 to 10) grades. Investigating the prevalence of fine codes by grade revealed that the most prevalent code in every grade was A1 (or Task Experts), and the prevalence of A1 increased from Grade 6 to Grade 10. In contrast, investigation into the variability of the prevalence of codes by class revealed considerable variations in all the codes from Grades 6 to 10 in each school. In particular, very large variations were noted for the prevalence of A1 by class in Grade 6.

Key Words: Decimal Comparison Test, Decimal Misconceptions, Task Experts, Prevalence, Variations.

Introduction

Decimals have become an issue after studies done indicating that some students cannot interpret and operate with decimals. Past research studies such as Grossman (1983), Sackur-Grisvard and Leonard (1985), and Moloney (1994) have found that the source of the issues with decimals came from the lack of initial understanding of decimal notation that stems from the teaching of decimals in primary schools.

Moloney and Stacey (1996) have stated that although decimals are first taught in primary schools, secondary students in many countries including Australia have inadequate understanding of the concepts within the nature of decimal notation. It was further stated, "Even some students who can calculate with decimals do not understand the comparative sizes of the numbers involved", (p. 4).

It has been well established by Steinle (2004) that one of the best ways to diagnose how students are thinking about decimal notation is by using a diagnostic test called the 'Decimal Comparison Test 2', hereafter, referred to as DCT2. The DCT2 contains 30 items where students are asked to choose the larger decimal number from pairs of decimals. The DCT2 has been administered to thousands of students from 12 Victorian schools between 1997 and 1999 as part of a study funded by the Australian Research Council (hence will be referred to as the ARC Study). The DCT1 (i.e. the first version of DCT2) was first used in the ARC Study from mid-1995 to 1996. However, after confirming that additional students' way of thinking could be reliably identified, a second version of the diagnostic test was then developed (hence DCT2).

Classes came from secondary schools (Grades 7 to 10) and their 'feeder' primary schools (Grades 4 to 6) representing a mix of various (high, medium and low) socio-economic groups and ages. The schools were grouped into six school groups (SGA to SGF). Students were given the same test approximately once every six months. In total there were 3204 students in Grades 4 to 10 completing 9862 tests between 1995 and 1999. It was from students' answers to the DCT2 that several decimal misconceptions were revealed.

The purpose of my study was to build on existing research about primary and secondary students' thinking about decimals. The analysis of my data has provided further evidence on the issue of the current understanding of decimal misconceptions raised following the study by Steinle (2004). One of the analyses investigated in my study was on the clustering of decimal misconception codes in classes and my aim was to establish variations by class across the primary (Grade 6 only) and secondary (Grades 7 to 10) grades.

The sample for my study was taken from 1998 and 1999, hence there were no new Grade 4 and Grade 5 students tested in 1998 and 1999. Students in Grade 4 and Grade 5 were last tested in 1996 and 1997 respectively. The decimal misconception code used in my study followed the coding used by Steinle (2004) and the description of these codes are provided in Table 1 (note that Table 1 below was taken from Table 2 of Stacey (2005, p. 24)).

Table 1: Matching of codes to the ways of thinking, from Stacey (2005), p. 24

Coarse Code	**Fine Code**	**Brief Description of Ways of Thinking**
A Apparent-expert	A1	Expert, correct on all items, with or without understanding.
	A2	Correct on items with different initial decimal places. Unsure about 4.4502/ 4.45. May only draw analogy with money. May have little understanding of place value, following partial rules.
L Longer-is-larger	L1	Interprets decimal part of number as whole number of parts of unspecified size, so that 4.63>4.8 (63 parts is more than 8 parts).
	L2	As L1, but knows the 0 in 4.08 makes decimal part small so that 4.7>4.08. More sophisticated L2 students interpret 0.81 as 81 tenths and 0.081 as 81 hundredths etc resulting in same responses.
S Shorter-is-larger	S1	Assumes any number of hundredths larger than any number of thousandths so 5.736<5.62 etc. Some place value understanding.
	S3	Interprets decimal part as whole number and draws analogy with reciprocals or negative numbers so 0.3>0.4 like 1/3>1/4 or -3>-4.
U Unclassified	U2	Can “correctly” order decimals, but reverses answers so that all are incorrect (e.g. may believe decimals less than zero).
	U1	Unclassified – not fitting elsewhere. Mixed or unknown ideas.

Cross-sectional prevalence of codes by grade

My study analysed 7887 tests that was collected between 1998 and 1999 from the ARC Study. Table 2 below contains the number of tests done in each grade of semesters 1 and 2 from the six school groups (note that Semester 1 and Semester 2 indicates the first half and the second half of the year (either 1998 or 1999) respectively).

Table 2: Number of tests in each semester by school group

Semester	School Group						Total
	SGA	SGB	SGC	SGD	SGE	SGF	
Gr6-Semester 1	45	30	95	93	43	0	306
Gr6-Semester 2	51	31	98	102	43	0	325
Gr7-Semester 1	127	163	547	353	149	152	1491
Gr7-Semester 2	85	307	275	161	73	0	901
Gr8-Semester 1	64	146	498	289	133	289	1419
Gr8-Semester 2	111	268	227	118	57	143	924
Gr9-Semester 1	110	114	244	138	138	282	1026
Gr9-Semester 2	93	205	0	0	66	141	505
Gr10-Semester 1	117	116	0	0	65	255	553
Gr10-Semester 2	116	175	0	0	0	146	437
Total	919	1555	1984	1254	767	1408	7887

A cross-sectional approach has been adopted which focused on the *tests* and not the students, indicating that in a period of one year a student might contribute either one or two tests. This method was to determine the total information (summative quantity) of each of the decimal misconception code in 1998 and 1999. The prevalence of the codes for each grade (Grades 6 to 10) were calculated, recorded and shown in Table 3 (expressed as a percentage). Note that A3, L4 and S5 are the unclassifieds A, L and S coarse codes respectively.

From Table 3, A1 was the most prevalent of all the fine codes. The prevalence of A1 increases from the younger to older students and Grade 10 has the highest prevalence of A1. Even though 70% of the tests from Grade 10 were A1, 30% were unable to answer the items in DCT2 correctly. This resulted in grouping 30% of the tests from Grade 10 to the other fine codes. For Grade 10 the next most prevalent code after A1 was U1 followed by A2 and then A3.

Table 3: Prevalence (%) of the fine codes by grade

Grades (No. of Tests)	A1	A2	A3	L1	L2	L4	S1	S3	S5	U1	U2
Gr 6 (n = 631)	52	2	3	10	5	3	5	5	2	14	0
Gr 7 (n = 2392)	57	4	3	7	4	2	4	4	3	13	1
Gr 8 (n = 2343)	66	4	3	2	2	1	2	6	2	10	2
Gr 9 (n = 1531)	68	5	3	3	2	1	2	4	1	10	1
Gr 10 (n = 990)	70	5	4	2	1	1	1	2	2	10	2
Overall (n = 7887)	63	4	3	4	3	1	3	5	2	11	1

Within the L behaviour in Table 3, a decreasing trend emerged across the grades for L1, L2 and L4. Grade 6 had the highest prevalence for all three L fine codes. The decreasing L trend showed that younger students (Grade 6 and Grade 7) tend to exhibit more L behaviour but eventually diminishes as students progressed to a higher grade. There is no much difference in the prevalence of S1, S3 and S5 in all grades. The S1, S3 and S5 trends seemed to be fluctuating between each grade. In total 866 tests (11%) were coded as U1, the next most prevalent code after A1. Grade 6 had the most U1 (14%) and Grade 8 to Grade 10 remained within the 10% mark.

Cross-Sectional Prevalence of Codes by Class

The next analysis of the cross-sectional study investigated the variability of the prevalence of the codes by class by means of clustering the codes that revealed decimal misconceptions into classes. The *prevalence of a code in a class* is calculated by dividing the number of tests allocated this code in a given class by the total number of tests completed in this class and expressed as a percent.

In total there were 219 classes (from Grades 6 to 10) involved in the overall analysis by class. Not all 219 classes have been tested twice in one year (some classes were only tested once in a year). In classes that were tested twice in one year, the tests from both semesters (i.e. Semester 1 and Semester 2) have been combined. There is the possible drawback of combining across semesters, i.e. the effect of recent teaching would be washed away. However, it is expected that this chosen analysis of clustering is less likely to pick up recent teaching effects; nevertheless it will provide a stronger data about systematic teacher effects, which would operate across the whole year.

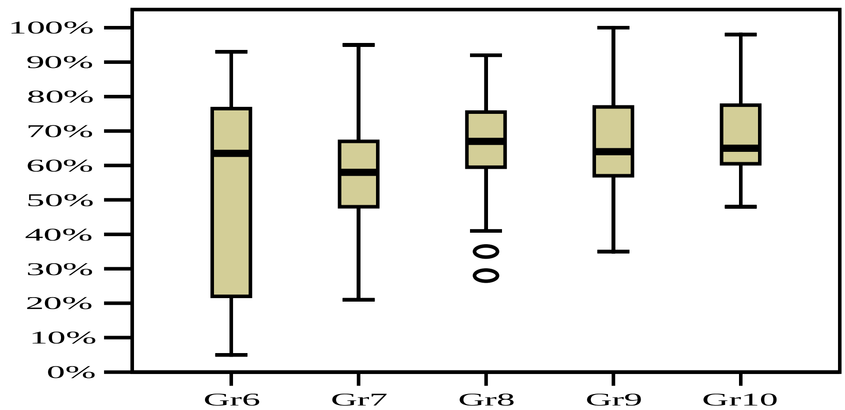
Initially (from the overall data), the number of tests in a class ranged from 4 to 60 tests (and the reason why 60 tests are large is due to the repeated testing in a year). However, a decision was made to exclude a class with less than 20 tests in my analysis to ensure that the overall results are not affected by small samples. Thus, the number of tests ranged from 20 to 60 in a class. Excel spreadsheets contributed to the box plots (for classes with more than 20 tests) to establish the variability of the prevalence of codes by class. The decimal misconception codes presented next will be limited to A2, L2, L3, S1 and S4 (i.e. these are the codes that revealed decimal misconceptions). However, A1 (i.e. the code that revealed *Task experts*) will also be included to show the variation it has revealed.

The percentage of classes that have at least one test coded as either one of the fine codes (limited to A1, A2, L2, L3, S1 and S4) from Grades 6 to 10 are presented in Table 4 respectively. For example, in Table 3 all 20 classes (i.e. 100%) in Grade 6 had tests coded A1; however the prevalence of A1 in each class varied. Another example from Table 3 is that from the 20 classes in Grade 6, 65% of these classes had tests coded L1 (however, the prevalence of L1 in each class varies) hence 35% of the Grade 6 classes had zero percent (or zero prevalence). Grade 6 classes with a zero prevalence indicate none of their tests was coded L1. In order to show how much the prevalence of a code by class varies, classes with zero prevalence have been included in the box plots.

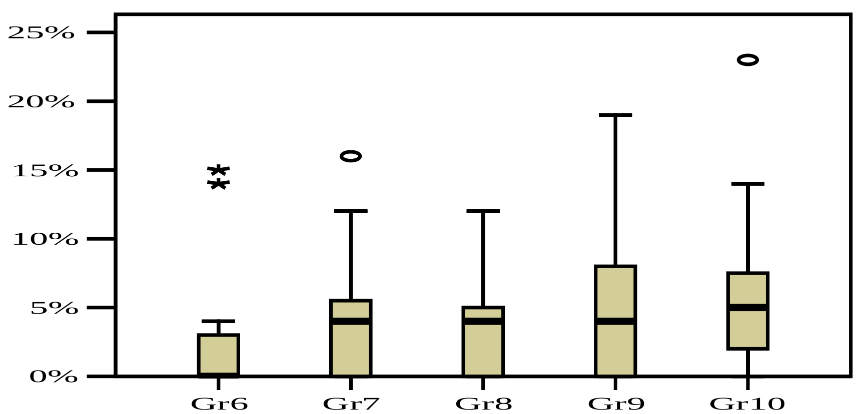
Table 4: Percentage of classes with fine codes within a grade

Grade (No. of classes)	A1	A2	L1	L2	S1	S3
Gr 6 (n = 20)	100	40	65	65	55	55
Gr 7 (n = 67)	100	70	72	66	70	70
Gr 8 (n = 63)	100	62	51	57	48	75
Gr 9 (n = 46)	100	65	41	33	46	72
Gr 10 (n = 23)	100	78	30	17	35	61
Overall (n = 219)	100	65	54	51	53	69

The variability of the prevalence of the codes (i.e. A1, A2, L2, L3, S1 and S4) for classes in Grades 6 to 10 are presented graphically in Figure 1 to Figure 3. The *median* (i.e. 50th percentile), *inter-quartile range* or *IQR* (i.e. the *box length* representing 25th and 75th percentile), *outliers* (indicated by a circle '○'), and *extremes* (indicated by an asterisk '*') of individual variables will be shown in each box plot. Outliers are defined as cases with values between 1.5 and 3 box lengths from the upper or lower edge of the box. Extremes are defined as cases with values more than 3 box lengths from the upper or lower edge of the box.

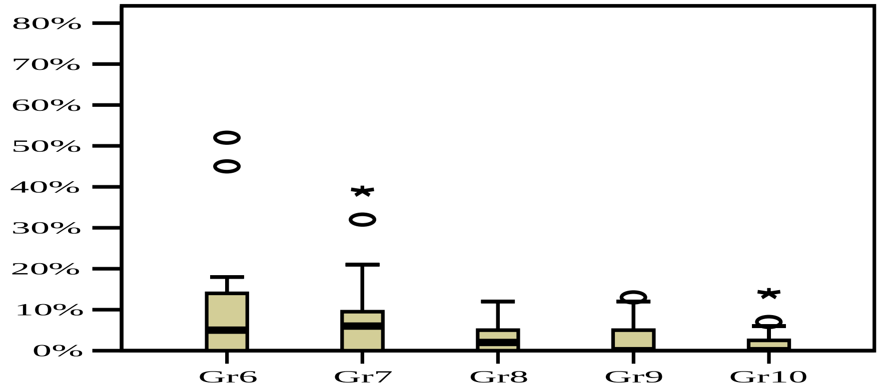


a) A1

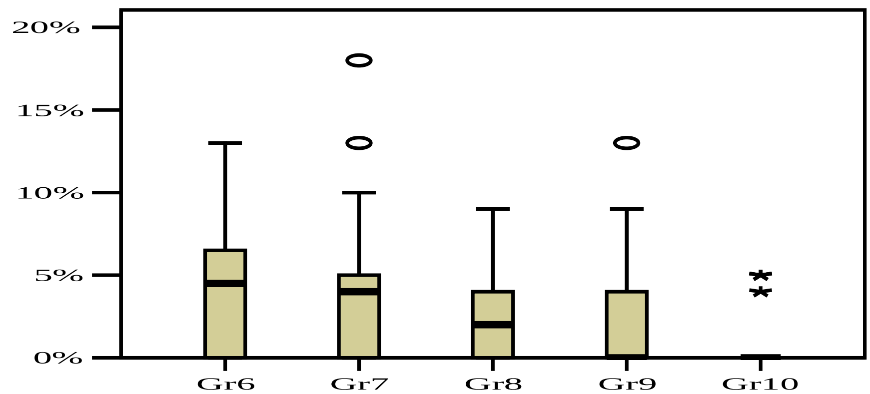


b) A2

Figure 1: Variability of prevalence of A1 and A2 by classes within a grade. (Note the vertical scales of A1 is different from A2)

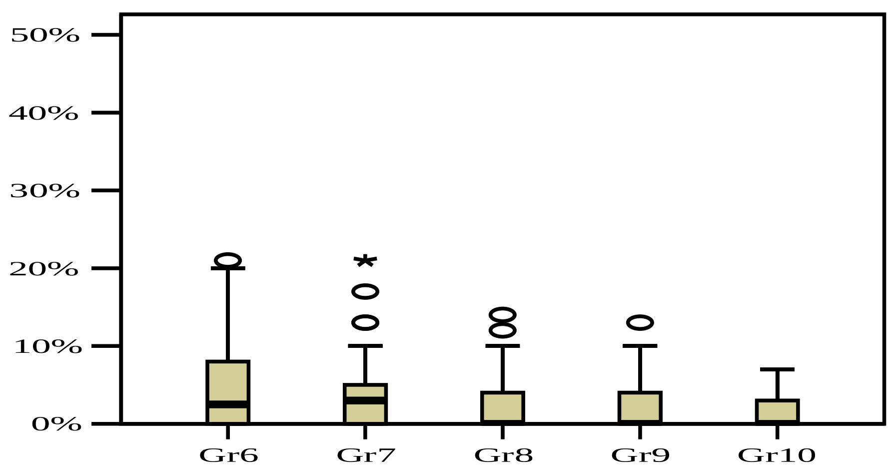


a) L1

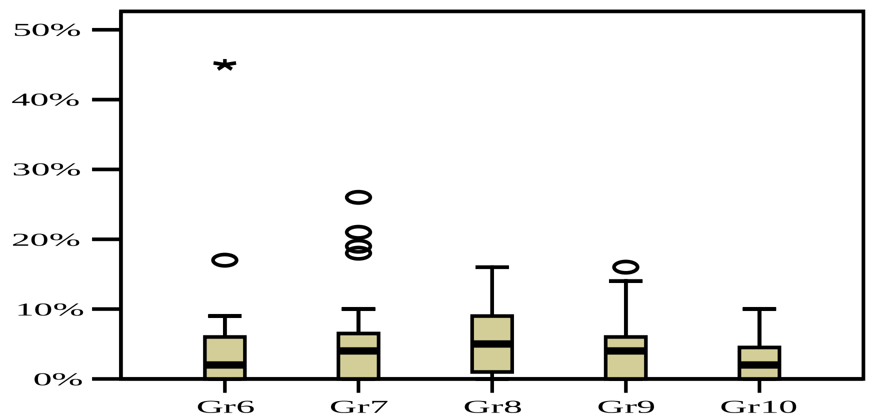


b) L2

Figure 2: Variability of prevalence of L1 and L2 by classes within a grade. (Note the vertical scales of L1 is different from L2)



a) S1



b) S3

Figure 3: Variability of prevalence of S1 and S3 by classes within a grade.

The Variability of the Prevalence of the Codes by Class

From Table 3, 52% of the Grade 6 tests were allocated the code A1. If teachers were to refer to Table 3, they might think that Grade 6 classes had an A1 average of 52%. However, I have shown here that there is a huge variation (i.e. to be anywhere between 5% and 93%) in each Grade 6 classes with tests coded A1 (refer to Figure 1a). To quote 52% as an average of Grade 6 tests coded as A1 is in fact not a good indication of the Grade 6 A1 representatives. In fact, careful analysis of the results showed that in each school, A1 representatives of Grade 6 varied across classes. Similarly for the variability of the prevalence of A1 for the other grades; the prevalence of A1 for Grade 7 was 57%, but the prevalence by class varied between 21% and 95%. Grade 8 and Grade 9 have almost similar results, but their variations of A1 by class ranged between 28% and 92% and, 35% and 100% respectively. The least variation was for Grade 10 (between 48% and 98%), the spread was not as large as the other grades (as shown in Figure 1a, the short box plot i.e. the IQR shows small variations between Grade 10 classes).

Figures 1 to 3 show that there are considerable variations in the prevalence of codes by class within a grade. With reference to Figure 1b, Grade 9 and Grade 10 had the most variation in the prevalence of A2. Beyond Figure 1, the IQR that showed the most variation were mostly for Grade 6 classes. In particular, the Grade 6 box lengths for L1 (0% to 14% in Figure 2a) and S1 (0% to 8% in Figure 3a). However for Grade 8, the IQR that showed variability was for S3 (i.e. between 1% and 9%). There were some codes that show a big difference (in the prevalence by class) between the extreme cases and the outliers. For example, in Figure 2a, the extreme case in the prevalence of L1 by class for Grade 6 was 52% in one class followed by 45% (indicated by the outlier) in another class. However, for Grade 7, the extreme was 39% followed by the outlier, 32%. In addition, for Grade 6, the extreme case in the prevalence of S3 (refer to Figure 3b) by class was 45% followed by the outlier, 17%. The difference in percentage between the extreme cases and the outliers showed that there were some low and some high variation from one class to the other (in terms of the decimal misconception their students had from the response of the DCT2).

Conclusion and Discussion

Investigating the prevalence of fine codes by grade revealed that the most prevalent code in every grade was A1, and the prevalence of A1 increased from Grade 6 to Grade 10. The significant rise in A1 over the grades was expected as students get older. The prevalence of L1, L2 and L4 decreased as students get older and the prevalence of L1 was the highest in Grade 6 (with 10%). The prevalence for S1, S3 and S5 however, were fluctuating between each grade. In contrast, investigation into the variability of the prevalence of codes by class revealed considerable variations when compared by class, in all the codes from Grades 6 to 10 in each school. In particular, very large variations were noted for the prevalence of A1 by class in Grade 6. While the average result for the prevalence of A1 in Grade 6 was 52%, the prevalence of A1 varied between 5% and 93% for individual classes.

Furthermore this was evident from Grade 6's A1 inter-quartile range i.e. IQR where very large variability was detected compared to the other grades. There were big differences in the prevalence of L1 and S3 by class between these two fine codes' extreme cases and outliers, in particular for Grade 6 and Grade 7. The IQR for S3 (between 1% and 9%) indicated there was variability in the prevalence of S3 for the majority of the Grade 8 classes.

So, why were there Variations?

The analysis of the prevalence of codes by class revealed considerable variations in most of the codes. Several hypotheses put forward here to explain "*why were there variations in the prevalence of A1 by class*" were; the teaching instruction of a teacher, classes with high prevalence of A1 had students retaining the teaching or the understanding of decimals well or it may also be the reasons of *project effect* where retesting improves performance. Since my data was obtained towards the end of the ARC Study (and majority of these students may have been tested and measured several times already earlier in the ARC Study), it was expected that repeated testing of students and the teachers' involvement in the study will increase the prevalence of Task experts (A1), hence the existence of project effect. Steinle (2004) defined the project effect to be "the improved performance on the DCT2 which was due to having completed the test before" (p. 95). Steinle (2004) estimated the magnitude of the project effect to be approximately 10%.

Steinle (2004) presented some possibilities about the effect of repeated testing amongst students that resulted in the increase of A1 (becoming experts). A summary of possibilities of the project effect from Steinle (2004, pp. 77-78) is as follows:

1) Students seeking additional help or paid more attention in future lessons, hence improving their knowledge on decimals and performing better by the next test.

2) A teacher who is motivated to increase the level of expertise in his/her class will focus on the task of decimal comparison (before the next round of testing) in one of the two ways:

> ***i.*** The 'good teaching' whereby a teacher, in an attempt to enhance students' conceptual understanding of decimals might focus on place value and the meaning of decimal numbers.
>
> ***ii.*** The 'superficial teaching' where a teacher introduces an algorithm to follow, allowing students to complete the test correctly without understanding why the algorithm actually works.

For my study, I do not know what the students' tests histories were, so, I can only assume the tests were from combination of 'old and new students' from the ARC Study. The 'old students' would have done the tests several times. Steinle (2004) reported on the testing of the project effect between "the proportions of students in a particular grade, who are experts in their first test, compared with other students in the same grade who are completing a subsequent test", (p. 78). She found the test effect to be positive for students in Grades 6 to 10 i.e. indicating "students who have done the test before are better than other students of the same age", (p. 78).
Steinle and Stacey (1998) concluded in their analysis of the A1 tests by school stating that "the number of A1 in a class will likely to be temporarily high if the testing took place immediately after instruction on decimal comparison", (p. 555). Consequently, this could also be a possibility which might contribute to the variations of the prevalence of A1 of my data.

Since evidence shown here that there were variations in the prevalence of codes by class, it seemed that one of the explanation could be the instruction taught by teachers. The teaching instruction on decimals has influenced students' thinking and their understanding of decimals, hence producing students with misconceptions. Moreover, in a study by Steinle and Stacey (1998) on the variability between classes by school, they concluded, "there also seems to be clear evidence that certain misconceptions are learned from school instruction. Some of the peaks may be temporary effects, so that students may be more likely, for example, to show a Shorter-is-larger misconception immediately after a unit on fractions", (p. 555). The overall conclusion was that students are doing decimal tasks with a lack of understanding of decimal notation. Students may have misconceptions and difficulties in interpreting and operating with decimals because their concept that underlies within the nature of decimal numbers has not been well developed.

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