A Critique of Tucker, a (1988) "Test of Black-Scholes and Constant Elasticity of Variance Currency Call Option Valuation Models"

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1.0 Problem statement, research objective, research questions, proposition and hypotheses

Tucker, (1988) research objective was to investigate empirically whether a constant elasticity of variance (CEV) currency call option valuation model demonstrates pricing accuracy superior to the extant currency call option pricing model (OPM). An adaptation of Ross and MacBeth call option valuation model for constant elasticity of variance diffusion process is tested against the adaptation of Black-Scholes call option valuation model for the pricing of call currency option.

2.0 Evaluation of Literature Review

An analysis of the efficiency of currency options markets and the pricing accuracy of currency option models by Bodurtha and Courtadon (6,7), Goodman, Ross and Schmidt (13), Johnson (15), Shastri and Tandon (22,23,24) and Tucker (26) was reviewed. The findings support market efficiency, but there was evidence by Bigger and Hull (4) of substantial mispricing in the European model. The mispricing was not attributed to misrepresentation of data or applying a European model with American data, but was attributed to the model misspecification since Black-Scholes solutionis deemed invalid even in a continuous limit when the underlying security price dynamics cannot be represented in a geometric Brownian motion.

MacBeth and Merville provide considerable evidence that returns on common stocks is heteroscedastic. The research indicates that the variance of stock returns and the stock prices levels are inversely related. The variance of the returns on foreign currency holding may also be related to prices. McFarland, Pettit and Sung found out that the distribution of daily exchange rate changes is too complex to be summarized by normal or stationary non-normal stable distribution. Metron (19) currency call option valuation model for continuous proportional dividends was analogous to the Black-Scholes model and was given by

$$c = e^{-r*T} SN(d_1) - e^{-rT} XN(d_1 - \sigma T^{.S}$$

d_1 = {ln(S/X) + [r-r^*+ ($\sigma^2/2$)] T/ $\sigma T^{.5}$}
c= the European call currency option price
S= the underlying spot exchange rate
X= the exercise price
T= time to maturity as a fraction of the year
r= domestic riskless rate of interest
r^*= the foreign riskless rate of interest
σ^2= instantaneous variance of the return on a foreign currency holding
N(.)= standard normal cumulative distribution function.

There was an assumption that the exchange rates followed a log normal diffusion process

dS/S= $\mu dt + \sigma\, dZ$,where
μ- mean exchange rate return,
t- time,
dZ- differential of a Weiner process.

Cox and Ross (10) proposed an alternative stochastic model to the log normal process to describe currency returns given by dS/S= $\mu dt + \delta\, S^{(\theta-2)/2} dZ$

δ= a scale coefficient
θ= the elasticity of the standard deviation of the exchange rate with respect to the exchange rate.

The above equation is a log normal process when θ=2 and $\sigma = \delta$ otherwise $\sigma = \delta\, S^{(\theta-2)/2}$. The model also allowed currency returns to be generated by a diffusion process where the standard deviation of returns varied with the exchange rate levels.

Direct estimation of the parameters of the CEV diffusion process was accomplished using the maximum likelihood estimation. Christie (8) approach was a derivation of the log likelihood function employed for maximum likelihood estimation for diffusion parameters δ and θ. The log likelihood function for a sample of size T was given by;

$$L = (-.5)(\ln(2\pi) + \ln(\delta^2)) + .5(2-\theta)\sum ln\,(s)\; - .5\delta^{-2}\sum r_s^2\, S^{2-\theta}$$

Where summations are over t= [1, T] and r_s = ΔS/S (ΔS- changes over discrete intervals). If θ=2, then the log normal holds.

3.0 Research Methodology

Tucker, 1988 adopted a methodology which was analogous to Emanuel and MacBeth (11) methodology. He calculated the root mean squared focused errors of observed currency call option prices for various option categories and prediction intervals. He compared predictive power by using model parameters estimated at time t which is the date of inception along with the exchange rate at time t+k for prediction of prices at the future date, t+k.

For the Black and Scholes model prediction for call option prices, the implied volatility of an at- the - money option on the inception rate, t is used as the estimate of σ for all trades occurring in time t+k. an implied standard deviation ($\hat{\sigma}_j$) for every six month quote for every currency was determined such that the model price equals the market price. The cross-sectional regression equation used was:

$$w_j\hat{\sigma}_j = \alpha + \beta\left[w_j\frac{S - Xe^{-rT}}{Xe^{-rT}}\right] + e_j$$

Where, w_j is square root of the number of contracts traded on each quote j and $\alpha + \beta$ are estimated intercepts and slope coefficients respectively.

α- represents an implied volatility of an exactly at- the- money six month currency call option. According to tucker, the method of generating α forces the Black and Scholes model to price correctly at- the- money six month options traded at their inception dates.

In the estimation of implied CEV model parameters δ *and* θ, the scale and elasticity coefficients of an at -the - money call options on the inception date t were used to establish model values of all trades occurring on date t+k. A value for the estimated elasticity coefficient for an exact at the money ψ was inferred since an option which is exactly at the money is not observed.

Estimates of the model parameters δ, θ *and* σ were also generated using historical exchange rate data. Exchange rates for every currency for the one hundred trading days immediately before every inception dates at time t was used to produce model parameters to predict option prices at time t+k.

3.1 Data used in Option Pricing Test

The data used for transactions currency call options was from the Philadelphia Exchange (PHLX). He omited the first nine months of trading so as to alleviate the cofounding seasoning effects. The data used was for over 30,000 quotes from September 19, 1983 through September 14, 1984. Each quote contained the time of trade as well as price and formulae data. He relaxes the assumptions of geometric Brownian motion while allowing the exchange rate dynamics to be represented stochastically in a continuous sample path.The PHLX had four inception dates for option traded during the time period of the research, i.e. September 19, 1983, December 19, 1983, March 19, 1984 and June 18, 1984. All the options initiated on these dates had three, six or nine months' maturities.

Tucker, 1988 focuses on the six months maturity. He excludes the three month maturities option on the basis that prices of short term options may not be sensitive to volatility, which may cause implied parameters to fluctuate substantially with small changes in price. Options with nine months maturities are excluded because of thin pricing.The unobservable model parameters used in the research on the six month currency call option were σ, δ *and* θ based on the observable parameters S, X, T r andr^{*}. Historical measures of σ, δ *and* θ were also used. Parameters used in comparing the predictive power of each model were $\sigma, \delta, \theta, X, r$ *and* r^{*} as of the date of inception, t, and the exchange rate S as of time t+k.

4.0 Summary of main findings and conclusions

Tucker, 1988 concludes that the CEV model predicts option prices better than the Black and Scholes model because it is more general and it allows currency return variance to be non-stationery. Also its MLE procedure indicates a direct and significant relationship between return variances and the exchange rates. For three or fewer trading days with stationery parameters, it also exhibits better pricing accuracy than Black and Scholes adaptation forced to price accurately six month at the money options traded on their inception dates.

5.0 An overall critique of the paper

Tucker, (1988) objective was to test the Black and Scholes model and the Constant Elasticity of Variance currency call options valuation models and to determine which gives accurate or better results. The paper wasn't well structured since the literature review was captured under the introduction part. It was difficult to ascertain which model was actually used in the analysis of data since Tucker kept on redefining the CEV and the BS models.A comparison of the two models was done using data from the Philadelphia Exchange where options were traded in five different currencies, namely British pound, Canadian Dollar, Deutsche mark, Japanese Yen and the Swiss Franc. Tucker, (1988) calculated the estimated elasticity coefficient$\hat{\sigma}$ and their standard errors. There was an indication that the elasticity coefficient was not significantly different from two (log-normality) at 5% level of confidence interval.The root mean forecast errors using the implied model parameters were calculated. A prediction interval of one, three and ten days was done both In-Money and Out-of-Money. In all the currencies, the standard errors for Black and Scholes were higher than the constant elasticity of variance under the three prediction intervals. From the results, one can observe that as the number of days increases under the prediction interval, the difference between the standard errors for Black and Scholes and CEV became smaller and smaller. This is an indication that the BS and CEV model standard errors will be almost similar as the number of days increases. Tucker should have extended the use of data for options which had more than six month's maturity, i.e. apply the concept to options that had nine months maturity. The practicability of CEV model is offset by the added complexities of implementation for all but short predictive horizons. Inter-temporally unstable elasticity coefficients have significant implications for valuation of international financial instruments other than currency options. The variables required for CEV model makes the practical application difficult, though models based on non-stationery processes still may apply.

6.0 Discussions of major extensions to the paper with specific suggestions about empirical methodology/ data for extension

Yang and Yuen, (1999) proposed a new estimation procedure for the CEV model which neither constraints on θ nor frequent estimations on the parameters was needed. They used the least squares estimation through which the two parameters δ *and* θ were estimated jointly. The merit of their model was that no constraints on the elasticity parameter of the models were imposed. Chan and Ng, (2006) developed a European option pricing formula for fractional market models. They argued out that although there existed option pricing results for a fractional Black-Scholes model, they are established without accounting for stochastic volatility. In their research, a fractional version of the Constant Elasticity of Variance (CEV) model was developed. European option pricing formula similar to that of the classical CEV model was obtained and a volatility skew pattern was revealed. Morozov, (2012) modelled elasticity of volatility as a stochastic process with an eye to merge popular constant elasticity of variance (CEV) and stochastic volatility (SV) models in order to understand when it is appropriate to use absolute or relative changes or some intermediate transformation as well as to compare with more traditional autoregressive exponential stochastic volatility formulations. In his research he described Markov chain Monte Carlo algorithm to efficiently sample from posterior distribution as well as associated particle filter for likelihood analysis and model comparison. His model showed tight yet strong variations in elasticities of interest rates variations.

Future work could be done by using different data sets and model comparisons via odds ratio, and extending the model to include fat-tailed distributions for measurement innovations as well as leverage. Fai lo, (2013)applied the Lie-Trotter operator splitting method to model the dynamics of both the sum and difference of two correlated constant elasticity of variance (CEV) stochastic variables. Within the Lie-Trotter splitting approximation, both the sum and difference were shown to follow a shifted CEV stochastic process, and approximate probability distributions were determined in closed form.These approximate probability distributions can be used to valuate two-asset options, e.g. the spread options, where the CEV variables represent the forward prices of the underlying assets. Aboulaich, Hadji and Jraifi, (2013) proposed an approximate numerical method for pricing of options for the constant elasticity of variance (CEV) diffusion model. They prove the existence and uniqueness of the solution in weighted Sobolev space, and proposed the finite element method and finite difference method to solve the considered problem. They compared the obtained results by the two approaches, with those given by the Monte Carlo method using two simulation techniques: the exact method and the Euler discretization. Their conclusions were that the two approaches were faster and accurate and are closer to the true market value.

Delbaen and Shirakwa studied the arbitrage free option pricing problem for constant elasticity of variance (CEV) model. Their intention was to find out the relationship between the CEV model and squared Bessel processes, to show the existence of a unique equivalent martingale measure, derive the Cox's arbitrage free option pricing formula through the properties of squared Bessel processes and to show that the CEV model admits arbitrage opportunities when it is conditioned to be strictly positive.

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